ELEMENTARY ALGEBRA

LONDON
Cambridge University Press
FETTER LANE

NEW YORK · TORONTO
BOMBAY · CALCUTTA · MADRAS
Macmillan

TOKYO
Maruzen Company Ltd

ELEMENTARY ALGEBRA

by

A. W. SIDDONS, M.A.
Late Fellow of Jesus College, Cambridge
Senior Mathematical Master at Harrow School

and

C. T. DALTRY, B.Sc.
Senior Mathematical Master at the Roan School, Greenwich

PART I

CAMBRIDGE
AT THE UNIVERSITY PRESS
1933

PRINTED IN GREAT BRITAIN BY W. LEWIS, M.A.
AT THE UNIVERSITY PRESS, CAMBRIDGE

PREFACE

Godfrey and Siddons' *Elementary Algebra* was published just over twenty years ago. The experience of those years has shown that the book may now be re-written with advantage. The present volume must be regarded as an entirely new work, but we have had the advantage of borrowing freely from the earlier book and we have retained its main features. One noteworthy change is the development of the old first chapter into Chapter 1, "The Use of Letters in Arithmetic", Chapter 2, "The Use of Letters in Geometry", Chapter 3, "Hidden Numbers—Equations", Chapter 4, "Problems".

The complete book will be divided into three parts of which the present volume is the first part. In this part there are developed the use of letters to represent numbers, the ideas of equations and problems (including of course much formula work), the manipulation of brackets, the ideas of graphs, the construction and reading of graphs, positive and negative numbers, fractions with very simple denominators.

Part II will include further work on formulae, simultaneous equations, factors, quadratics and further work with graphs and fractions.

Part III will include logarithms, variation, progressions, the binomial theorem, the elements of calculus and other matter.

The authors are greatly indebted to many who have used Godfrey and Siddons' *Elementary Algebra* for suggestions and criticism and also to many discussions in committees and meetings in which they have taken part.

The authors' thanks are also due to the Controller of H.M. Stationery Office, the Oxford and Cambridge Joint Board, London University, the Cambridge Local Examination Syndicate, the

Oxford Local Examination Delegacy, the Northern Universities Joint Matriculation Board, the University of Bristol and the Board of the Common Entrance Examination for Public Schools for permission to include questions from various public examinations.

A. W. S.

C. T. D.

May 1933

TABLE OF CONTENTS

TO THE TEACHER

Exercises which are numbered (i) and (ii), e.g. Exercise 3 *d* (i) and Exercise 3 *d* (ii), are parallel exercises: it is intended that they should be used in alternate terms or in case exceptional difficulty has been found.

Answers that can easily be found at sight are not given at the end of the book.

The teacher will find it helpful to write the answers to oral exercises beside the questions; this will enable him to watch his class more closely.

Teachers are strongly advised to make pupils keep all the graphs that they draw; many of the graphs given in the chapters on graphs are required again after the place at which they are drawn. It is recommended that the graphs should be drawn in a special notebook, or on loose sheets of paper that can be stuck into a notebook: it is only necessary to gum a narrow strip at the edge of the paper.

CHAPTER 1

THE USE OF LETTERS IN ARITHMETIC

1·1. Algebra may be regarded as an extension of Arithmetic.

In Arithmetic we study processes and problems involving numbers.

Algebra begins with the study of similar processes and problems in which letters, a, c, x, A, V, ..., are used to represent numbers.

Before anything really new can be done in Algebra, it is necessary for us to handle letters as confidently as we handle numbers. To acquire this confidence we will revise some of the processes of Arithmetic, using letters instead of numbers. In this work we shall be interested in the essence of the processes instead of in the numerical results.

CHANGE OF UNIT

1·2. Example. Shillings and pence.

$$\begin{array}{llll} 1 \text{ shilling} & = 12 \text{ pence}, & 150 \text{ pence} & = \dfrac{150}{12} \text{ shillings}, \\ 3 \text{ shillings} & = 12 \times 3 \text{ pence}, & & \\ s \quad ,, & = 12 \times s \quad ,, & p \quad ,, & = \dfrac{p}{12} \quad ,, \end{array}$$

Copy the following table; fill up the gaps, but do not do any multiplications or divisions (e.g. write 17×12 and do not go on to 204, write $\dfrac{108}{12}$ and do not go on to 9):

Number of shillings	1	2	3	17	s				
Number of pence						12	24	108	p

We recall the rule: "To change a number of shillings into pence we multiply the number of shillings by 12".

* This chapter suggests work that could well be done in Arithmetic, thus paving the way for Algebra as well as bringing out and summing up the essentials of the arithmetical processes. If this has not been done in the Arithmetic course, it should be treated fully at the beginning of the Algebra course.

We can state this in symbols,

$$s \text{ shillings} = s \times 12 \text{ pence.}$$

Now state in words and in symbols the rule for changing a number of pence into shillings.

Example. Suppose 1 pound $= f$ francs.

Then 3 pounds $= f \times 3$ francs

and p pounds $= f \times p$ francs.

NOTATION FOR MULTIPLICATION

1·3. The signs $+$, $-$, $\times$, $\div$ are used in Algebra just as in Arithmetic; but "a divided by b" is usually written as $\frac{a}{b}$ and not $a \div b$.

It is usual to abbreviate in Algebra by leaving out multiplication signs. Thus we abbreviate $12 \times s$ to $12s$, and $f \times p$ to fp.

But notice that 2×3 must not be written 23 (for this means twenty-three); we must leave it as 2×3 or else write 6.

You know that $4 \times 5 = 5 \times 4$, that $12 \times 16 = 16 \times 12$, and in fact that when any two numbers are multiplied together, it does not matter which is multiplied by which.

Now a letter in Algebra stands for a number; hence $s \times 12 = 12 \times s$, and each of these **expressions** is written $12s$. The number is always placed first.

Exercise 1 a (Oral)

1. How should you write, without multiplication signs, $p \times 4$, $4 \times p$, $20 \times a$, $a \times 20$, 20×2, $a \times x$, $x \times a$?

2. Why is it wrong to write 202 for 20×2?

3. What is meant by $7b$?

4. Put into words the statement that $a \times b = b \times a$ whatever numbers a and b stand for.

Exercise 1 b (Oral)

1. Fill up the gaps in the following table:

Number of feet	1	2	4	13	f	t
Number of yards						

State (i) in words, (ii) in symbols, what you do to change from feet to yards, and what you do to change from yards to feet.

2. Fill up with numbers and symbols the gaps in the following table:

Number of feet						
Number of inches						

State (i) in words, (ii) in symbols, what you do to change from feet to inches, and what you do to change from inches to feet.

3. Make a table showing the number of pounds equivalent to various numbers of shillings. State in words and in symbols what you do to change from shillings to pounds, and to change from pounds to shillings.

4. Similarly state in words, and then in symbols, using the letters indicated, the connection between

(i)	Pence and pounds	d, p.
(ii)	Yards and miles	y, m.
(iii)	Yards and inches	y, i.
(iv)	Pounds and ounces	l, z.
(v)	Pounds and tons	l, t.
(vi)	Minutes and hours	m, h.
(vii)	Francs and centimes	f, c.
(viii)	Pints and gallons	p, g.

FRACTIONS

1·4. In Arithmetic we learnt the golden rule for fractions:

The value of a fraction is unaltered by multiplying (or dividing) both its numerator and its denominator by the same number.

Thus $\frac{2}{3}=\frac{2\times 2}{3\times 2}=\frac{4}{6}$; similarly $\frac{2}{3}=\frac{2a}{3a}$ and $\frac{a}{b}=\frac{ax}{bx}$

Again $\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$; similarly $\frac{2b}{3b} = \frac{2}{3}$ and $\frac{cy}{dy} = \frac{c}{d}$.

Also $\frac{3}{4} \times 5 = \frac{3 \times 5}{4}$; similarly $\frac{3}{4} \times n = \frac{3 \times n}{4} = \frac{3n}{4}$ and $\frac{p}{q} \times n = \frac{pn}{q}$.

We have learnt in Arithmetic that

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5};$$

similarly

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}.$$

Also

$$\frac{5}{2} \div \frac{4}{3} = \frac{\frac{5}{2}}{\frac{4}{3}} = \frac{\frac{5}{2} \times 2 \times 3}{\frac{4}{3} \times 2 \times 3} = \frac{5 \times 3}{4 \times 2};$$

similarly

$$\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b} \times b \times d}{\frac{c}{d} \times b \times d} = \frac{ad}{bc}.$$

EXERCISE 1 c (Oral)

1. Simplify:

(i) $\frac{3 \times 2}{5 \times 2}$, (ii) $\frac{3a}{5a}$, (iii) $\frac{ca}{ba}$, (iv) $\frac{3ca}{5ba}$,

(v) $\frac{6pq}{10qr}$, (vi) $\frac{6x}{10}$, (vii) $\frac{4}{6y}$, (viii) $\frac{2r}{r \times r}$.

2 Fill in the gaps in

(i) $\frac{4q}{6} = \frac{\quad}{3}$, (ii) $\frac{10ab}{15ac} = \frac{2b}{\quad}$, (iii) $\frac{ar}{rb} = \frac{\quad}{b}$.

3. Write down three fractions equivalent to $\frac{12ab}{18ac}$; three equivalent to $\frac{x}{3}$; and three equivalent to $\frac{2}{n}$.

4. Simplify:

(i) $\frac{3}{4} \times 12$, (ii) $\frac{3}{4} \times 4a$, (iii) $\frac{3}{a} \times 4a$, (iv) $\frac{x}{a} \times 4a$.

5. Simplify:

(i) $\frac{4}{5}\times\frac{1}{2}$, (ii) $\frac{4}{5}\times\frac{3}{2}$, (iii) $\frac{4}{a}\times\frac{b}{2}$, (iv) $\frac{x}{a}\times\frac{a}{y}$, (v) $\frac{x}{a}\times\frac{a}{x}$.

6. Simplify:

(i) $\frac{2}{3}\div\frac{1}{6}$, (ii) $\frac{2}{3a}\div\frac{1}{a}$, (iii) $\frac{2a}{5}\div\frac{a}{10}$, (iv) $\frac{a}{c}\div\frac{b}{c}$.

7. How many 2*d.* stamps can be bought with s shillings?

8. A bus-fare is 3*d.* How many rides can be taken with c half-crowns?

9. A sheet of 1*d.* stamps has n rows containing 6 stamps each. Find its value in shillings.

10.

Change to	pence,	$4f$ farthings,	$6x$ halfpennies.
,,	shillings,	$24p$ pence,	$8y$ sixpences.
,,	pounds,	$2h$ half-crowns,	$3f$ florins.
,,	yards,	$27f$ feet,	$14n$ inches.

11.

Change to	shillings,	$\frac{l}{4}$ pounds,	$\frac{h}{2}$ half-crowns.
,,	tons,	$\frac{c}{8}$ cwt.,	$\frac{112}{g}$ lb.
,,	pounds,	$10z$ oz.,	$\frac{k}{14}$ cwt.
,,	feet,	$\frac{3y}{4}$ yards,	$\frac{48a}{5}$ inches.

12. Simplify *where possible*:

(i) $\frac{2a}{3a}$, (ii) $\frac{ax}{bx}$, (iii) $\frac{ay}{bx}$, (iv) $\frac{a+2}{a}$, (v) $a\times\frac{2}{a}$,

(vi) $\frac{a+b}{b}$, (vii) $\frac{ab}{b}$, (viii) $\frac{a-b}{ab}$, (ix) $\frac{a-x}{b-x}$, (x) $\frac{xy}{yx}$.

UNITARY METHOD AND ITS EXTENSIONS

1·5. Example. If 3 books cost 5 shillings,

then 1 book costs $\frac{5}{3}$ shillings.

Similarly, if b books cost s shillings,

1 book costs $\frac{s}{b}$ shillings.

Example. If b books cost p pence, what is the cost in pence of k books?

b books cost p pence,

$\therefore$ 1 book costs $\frac{p}{b}$ pence,

$\therefore$ k books cost $\frac{p}{b} \times k$ pence.

Or, if we leave out the unit line,

b books cost p pence,

$\therefore$ k books cost $p \times$ — pence.

We know that we have to multiply by b or k and divide by the other. Suppose that k is larger than b, will the cost be more or less than p pence? Certainly more. Then multiply by k and divide by b.

Consider this again, assuming that k is smaller than b.

Example. If p men can do a piece of work in d days, how long will q men take?

p men can do it in d days,

q ,, ,, $d \times$ — days.

Consider (i) when q is greater than p, (ii) when q is less than p.

Exercise 1 d

Many of the following should be discussed orally before they are written out. If difficulty is found with any example, a numerical case should be discussed first.

1. A pencil costs p pence. What do a dozen pencils cost (i) in pence, (ii) in shillings?

2. 1 lb. of tea costs s shillings. What do l lb. cost (i) in shillings, (ii) in £?

3. z oz. of tobacco cost t pence. What is the cost of 1 lb. (i) in pence, (ii) in shillings?

4. a feet of elastic cost b pence. What do c feet cost?

5. A clerk addresses n envelopes in an hour. How many hours will he take to address N?

6. The cost of x tons of coal is £P. How many tons can be bought for £Q?

7. There are c centimetres in 1 inch. How many metres are there in 1 yard?

8. If you walk at the rate of 3 miles an hour, how far will you walk in 2 hours? How far in x hours?

9. What distance can a train go in y hours at the rate of 60 miles an hour?

10. A train travels z miles in t hours. What is its speed in miles per hour?

11. A train travels d miles at 60 miles per hour. How long does it take?

12. A train travels m miles in h hours. (i) How far will it go in k hours? (ii) How long will it take to travel s miles?

13. A train travels at v miles per hour. How many feet per second is this?

14. Complete the spaces in the following tabulated sums:

		Weight	Cost
(i)		p lb.	s shillings
	$\therefore$		h shillings
(ii)		z oz.	q pence
	$\therefore$	Z oz.	
(iii)		a cwt.	£L
	$\therefore$	b tons	

15. Make up for yourself three unitary method sums, choosing your own symbols.

LENGTHS, AREAS, VOLUMES

1·6. Fig. 1·1 shows a length of wire bent into the shape of a rectangle. The length of wire used to make this shape is the sum of two pieces l in. long and two pieces b in. long, i.e.

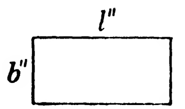


Fig. 1·1

$$\text{Total length} = l \text{ in.} + l \text{ in.} + b \text{ in.} + b \text{ in.}$$

By inserting brackets we can abbreviate this, to read

$$\text{Total length} = (l + l + b + b) \text{ in.}$$

The contents of the bracket are regarded as a single number, and give

$$\text{Total length} = (2l + 2b) \text{ in.}$$

or

$$= (2b + 2l) \text{ in.}$$

To find the total length we have to add the **expression** $2b$ to the expression $2l$. We speak of $2b$ and $2l$ as **terms** of the expression $2l + 2b$.

As we cannot simplify the sum $2l + 2b$, the terms $2l$ and $2b$ are called **unlike terms**. By contrast, $l + 2l$ can be written as $3l$: we say that in $l + 2l$, the l and $2l$ are **like terms**.

Example. To form the wire frame in fig. 1·2 we require a length of

$$(l + l + l + b + b + b) \text{ in.}$$
$$= (3l + 3b) \text{ in.}$$

1·7. Again from our Arithmetic* we see that the area of the rectangle in fig. 1·3 is $l \times 2$ sq. in. or $2l$ sq. in.; also the area of the rectangle in fig. 1·2 is $l \times b$ sq. in. or lb sq. in.

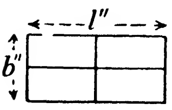


Fig. 1·2

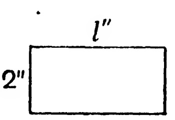


Fig. 1·3

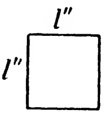


Fig. 1·4

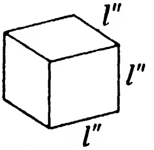


Fig. 1·5

If a rectangle has two adjacent sides equal, it is called a square, and its area is $l \times l$ sq. in., which may be written ll sq. in. or l^2 sq. in.

If we consider a solid cube (see fig. 1·5) each of whose edges is l in. long, the total surface area of the cube is $l \times l \times 6$ sq. in., i.e. $6l^2$ sq. in.

* It may be well at this stage to revise the fundamental idea of area as developed in Arithmetic.

In this expression notice particularly that only the l is squared. If we wanted both the 6 and the l to be squared we should write $(6l)^2$, meaning $(6l) \times (6l)$, i.e. $6 \times l \times 6 \times l$, i.e. $36l^2$.

Example. Fig. 1·6 shows the cross-section of a hollow iron bar. The area of the section

$$= (2s)^2 - s^2 \text{ sq. in.}$$
$$= 2s \times 2s - s^2 \text{ sq. in.}$$
$$= 4s^2 - s^2 \text{ sq. in.}$$
$$= 3s^2 \text{ sq. in.}$$

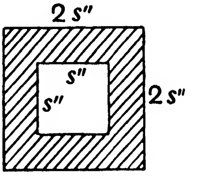


Fig. 1·6

Consider the closed box shown in fig. 1·7.
The end is a square of side s in.
Then the total length of edge of the box is

$$(4s + 4s + 4l) \text{ in.,}$$
$$= (8s + 4l) \text{ in.}$$

The total surface area is

$$(s^2 + s^2 + 4sl) \text{ sq. in.,}$$

i.e. $$(2s^2 + 4sl) \text{ sq. in.}$$

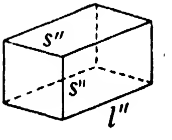


Fig. 1·7

The volume is $s \times s \times l$, i.e. $s^2 \times l$ cu. in.
We abbreviate $s^2 \times l$ to s^2l.
Since $s^2 \times l = l \times s^2$, s^2l may also be written ls^2.
In each of these expressions for the volume note that only the s is squared.

1·8. Fig. 1·5 shows a wooden cube.

(i) The total length of edge $= 12l$ in.

(ii) The total area is 6 times the area of a face, i.e. is

$$6 \times l^2 \quad \text{or} \quad 6l^2 \text{ sq. in.}$$

(iii) The volume is $l \times l \times l$ cu. in., which is written l^3 cu. in.

If the edge of the cube had been $2l$ in., then the volume would have been $(2l)^3$ cu. in.,

i.e. $$(2l) \times (2l) \times (2l) \text{ cu. in.}$$
$$= 2 \times l \times 2 \times l \times 2 \times l \text{ cu. in.}$$
$$= 8l^3 \text{ cu. in.}$$

You should try to see this from fig. 1·8.

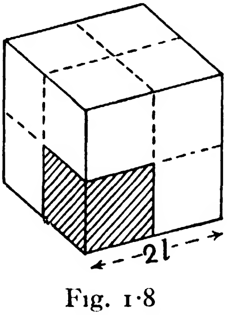


Fig. 1·8

Exercise 1 e

1. Write in their simplest form

(i) $a \times 2 \times b$, (ii) $l \times b \times h$, (iii) $a \times a \times 5$,

(iv) $2a \times 2a$, (v) $(3a)^2$, (vi) $a \times a \times 4a$,

(vii) $(2a)^3$, (viii) $(2a)^2 \times h$, (ix) $w \times s \times (2s)^2$.

2. Simplify:

(i) $2s+3-s$, (ii) $a+2b+2a+1$, (iii) $2x+y+3y-x$.

3. Which of the following can be simplified?

(i) $a+b+a$, (ii) $ab+ab+ba$, (iii) $ab+1$,

(iv) $ab-a$, (v) $ab+a+b$, (vi) $1-a+b-1$.

4. When $s=3$ and $t=2$, which of the following is the greatest? s^2, $2t^2$, $3st$, $s-t+st$, $3s-4t$, $(2t)^2$.

5. What is the difference between $2t^2$ and $(2t)^2$ when $t=3$?

6. Make a freehand sketch of each of figs. 1·9 to 1·13. Find (i) the perimeter, (ii) the area of each figure.

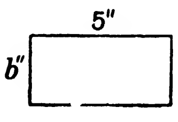


Fig. 1·9

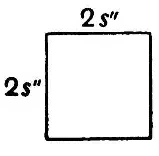


Fig. 1·10

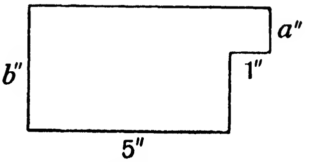


Fig. 1·11

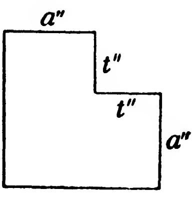


Fig. 1·12

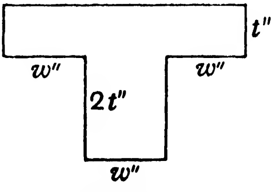


Fig. 1·13

7. Find the area of the unshaded parts of figs. 1·14–1·16 (i) by considering the difference between the areas of the

whole figure and the shaded part, (ii) by aid of the broken lines.

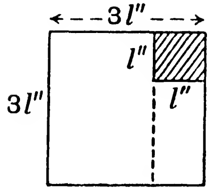


Fig. 1·14

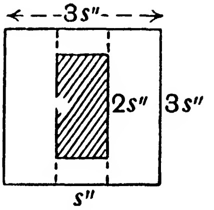


Fig. 1·15

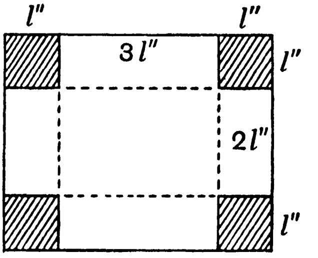


Fig. 1·16

8. Find (i) the surface area, (ii) the volume, of each of the solids in figs. 1·17–1·20.

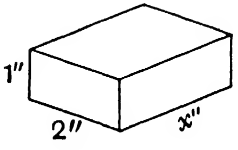


Fig. 1·17

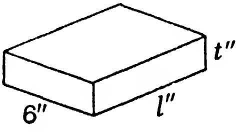


Fig. 1·18

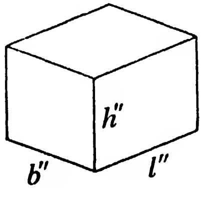


Fig. 1·19

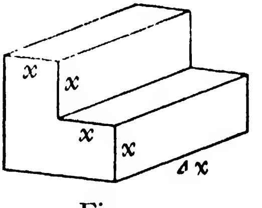


Fig. 1·20

FURTHER EXAMPLES

1·9. It is not intended that the following examples should be taken at this stage, but it is suggested that they should be worked with the corresponding examples in Arithmetic.

Exercise 1 f

1. What is 1 % of n?

2. What is c % of p?

3. If a % of a number is b, what is the number?

4. $x\%$ of a number is N. What is $y\%$ of the number?

5. What per cent. is 1 of 2,
3 of 5,
a of b,
1 of X?

6. On a cost price of s shillings I make a gain of g shillings. What is my gain per cent.?

7. Fill in the gaps.

Cost Price	Selling Price	Gain	Gain %
c	s		
p		g	
	x	y	
a			b
		q	r
	s		t

8. By selling an article for £P, I gained 5 %. What did the article cost me?

9. $g\%$ is gained by selling at S pence. What percentage is gained by selling at P pence?

10. Divide a legacy of £L in the ratio $a : b : c$. Can we say which share is the greatest?

11. Two powders are mixed in the ratio $p : q$ by weight. How much of each is there in l lb. of the mixture?

12. What is the average of a, b, c?

13. The average age of p boys is a years, the average age of another q boys is b years; what is the average age of the whole lot?

14. A can do a piece of work in x days, B can do it in y days; how many days will they take if they work together?

CHAPTER 2

THE USE OF LETTERS IN GEOMETRY

2·1. In this chapter we assume a knowledge of the facts about

(i) angles at a point,

(ii) angles formed by parallel lines and a transversal,

(iii) the angle-sum of a triangle and a polygon,

(iv) the base angles of an isosceles triangle.

We indicate a right angle as in fig. 2·1, and a pair of parallel lines by arrowheads as in fig. 2·2.

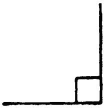


Fig. 2·1

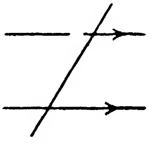
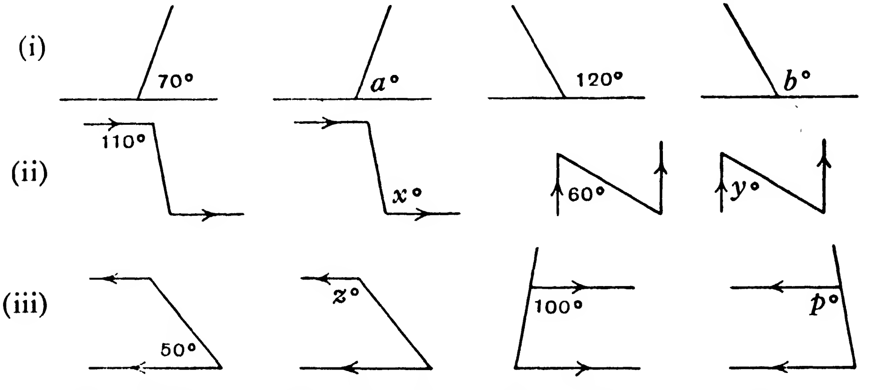


Fig. 2·2

The marks on the lines in the figure of Ex. 2*a*, No. 1 (v) indicate that the lines are equal.

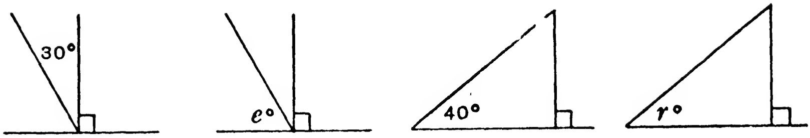
Exercise 2 a (Oral)

1. Find the sizes of the unmarked angles in these figures:

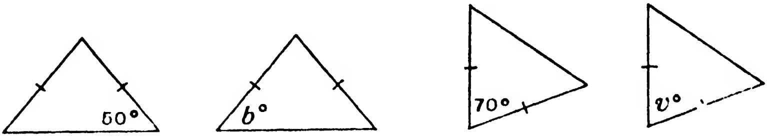


* This chapter suggests work that could be done in the Geometry course; here it may be used to generalise the Geometry and to pave the way for Algebra. If Geometry has not yet been begun, the chapter should be omitted.

(iv)

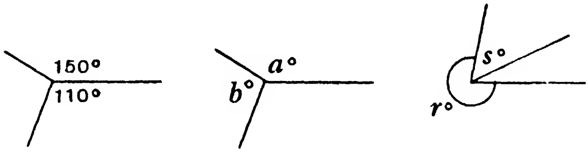


(v)

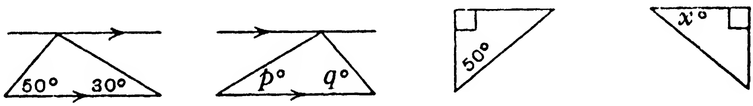


2. Find the sizes of the unmarked angles in these figures:

(i)



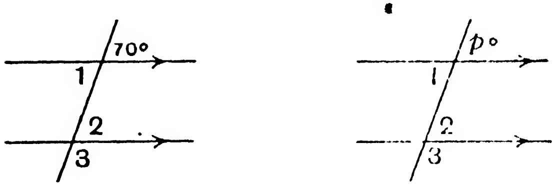
(ii)



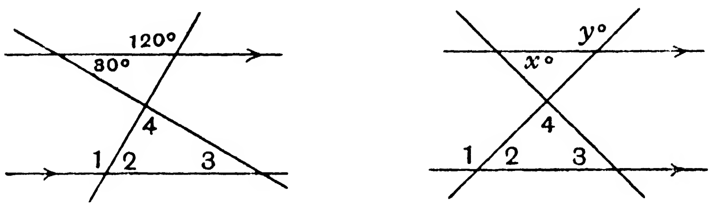
(iii)

3. Calculate the size of the numbered angles in these figures:

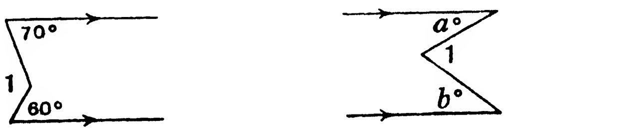
(i)



(ii)



(iii)



4. If x° and y° are two supplementary angles, write down the connection between x and y.

5. If the angles of a triangle are A°, B°, C°, write down a fact about A, B, C.

6. If the three angles of a triangle are 90°, r° and s°, what follows about r and s?

7. In fig. 2·3, prove $x=y$.

8. In fig. 2·4, prove $p+q=r$.

9. In fig. 2·5, prove $s=360-r-t$.

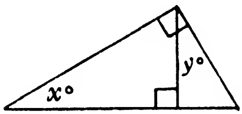


Fig. 2·3

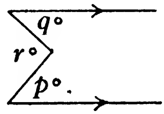


Fig. 2·4

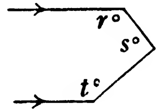


Fig. 2·5

10. In fig. 2·6, prove $\angle$PQR$=90^\circ$.

11. In fig. 2·7, prove $p=90-\dfrac{q}{2}$.

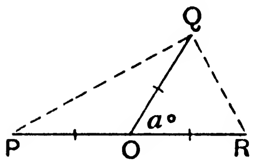


Fig. 2·6

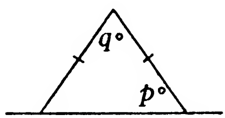


Fig. 2·7

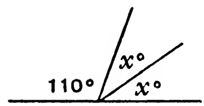


Fig. 2·8

12. Find x (i) in fig. 2·8, (ii) in fig. 2·9, (iii) in fig. 2·10.

13. Find x, y, z in fig. 2·11.

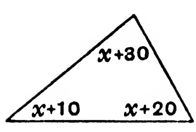


Fig. 2·9

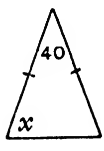


Fig. 2·10

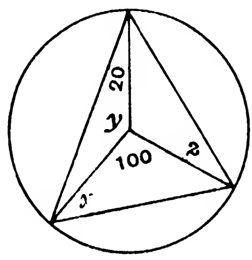


Fig. 2·11

CHAPTER 3

HIDDEN NUMBERS—EQUATIONS

3·1. Algebra began when the Arabs discovered the use of equations for solving problems that appeared too hard for ordinary arithmetical methods.

This chapter shows how equations can be so used.

3·2. Problem. Think of a number, double it, add 7, suppose that you now tell me that the result is 25. I will find what number you thought of.

Suppose that the number you thought of is written on the blackboard, concealed behind a piece of paper; then the facts may be written thus:

$$2 \times \boxed{} + 7 = 25$$

I have to find what is the number behind the paper.

The two sides of this **equation** are equal; and if I subtract 7 from each side, the sides are still equal.

I then have

$$2 \times \boxed{} = 18$$

If I divide each side by 2, the sides remain equal; then

$$\boxed{} = 9$$

which means that the number behind the paper is 9.

Now see what $2 \times 9 + 7$ is equal to; is it 25?

Instead of pinning up a piece of paper in front of the hidden number, I will suppose that n stands for this number; then the equation will be

$$2 \times n + 7 = 25,$$

or

$$2 \cdot n + 7 = 25,$$

and to **solve** this equation (i.e. to find the number for which n stands) the steps would be as follows:

Subtract 7 from both sides, $\therefore\ 2 \times n = 18.$

Divide both sides by 2, $\therefore\ n = 9.$

Check. $2 \times 9 + 7 = 18 + 7 = 25.$

Exercise 3 a (Oral)

In the following equations the letters stand for hidden numbers; find the numbers:

1. $n+7=10.$	**2.** $5+p=12.$	**3.** $q-12=6.$
4. $12-r=6.$	**5.** $3s=18.$	**6.** $15=5t.$
7. $u-8=0.$	**8.** $0=3v-18.$	**9.** $13=4+w.$
10. $3=7-x.$	**11.** $3=y-7.$	**12.** $13=z+8.$
13. $1+a=1\frac{1}{2}.$	**14.** $3-b=2\frac{3}{4}.$	**15.** $\frac{1}{2}+c=7.$

3·3. In most of the above examples we have been able to guess the hidden number more or less without any method, but to solve harder examples we must develop definite methods.

Consider $3n+4=40.$

This is a statement that two numbers are equal.

For a moment let us think of a balance, a pair of scales.

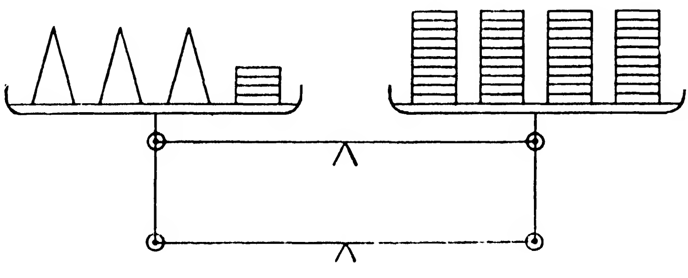


Fig. 3·1

Suppose the left-hand pan contains three lumps of lead, indistinguishable from one another, and also 4 one-ounce weights; suppose the other pan contains 40 one-ounce weights; and suppose that the two sides balance. Should we upset the balance if we took 4 one-ounce weights from each pan?

In the same way we see that if we take 4 from each side of our equation, then $3n=36$.

Suppose we now emptied our scale-pans and put back one lump of lead in the left pan, and one-third of the 36 one-ounce weights in the right. Should we expect them to balance?

In the same way if we divide $3n$ by 3 and 36 by 3 we should expect the results to be equal.

$$\therefore \frac{3n}{3}=\frac{36}{3},$$

i.e.

$$n=\frac{36}{3}=12.$$

We verify, or check, this answer by writing 12 instead of n in the left-hand side of the original equation, $3n+4=40$.

We find $3n+4=36+4=40$, as it should.

Hence the solution $n=12$ is correct.

3·4. Again, suppose a pair of scales is balancing with two different looking loads, one in each pan.

We should say

Weight in one pan = Weight in the other pan.

The balance would not be upset if equal weights were added to, or taken from, each pan.

Now an equation is a sort of mathematical balance in which numbers are compared instead of weights.

You should see that

(i) **if the same numbers are added to each side of an equation, the two sides will again be equal;**

(ii) **if the same numbers are taken away from each side of an equation, the two sides will again be equal.**

Note this will be true whether the numbers added or subtracted are known numbers or unknown numbers; all that is essential is that they should be equal.

In the same way keeping the idea of a pair of scales before us we can see that

(iii) **if each side of an equation is multiplied by the same number, the two sides will again be equal;**

(iv) **if each side of an equation is divided by the same number, the two sides will again be equal.**

Here also the multipliers (or the divisors) may be known or unknown numbers; all that is essential is that they should be equal.

3·5. Example. Solve $47-3n=7n+12$.

Add $3n$ to each side.

[As n represents a number, so does $3n$. Hence we are adding the same number to each side, though it is an unknown number.]

$$\therefore\ 47=7n+12+3n.$$

Subtract 12 from each side,

$$\therefore\ 35=10n.$$

Divide each side by 10,

$$\therefore\ 3\cdot5=n,$$

i.e. $$n=3\cdot5 \text{ or } 3\tfrac{1}{2}.$$

Check. When $n=3\frac{1}{2}$,

$$\text{left-hand side} =47-10\tfrac{1}{2}=36\tfrac{1}{2},$$

$$\text{right-hand side}=24\tfrac{1}{2}+12=36\tfrac{1}{2}.$$

We abbreviate left-hand side to L.H.S. and right-hand side to R.H.S.

EXERCISE 3 b

Solve the following equations. Explain every step of your work, and check your answers.

Nos. 1–12 should be discussed orally.

1. $2n+1=35$. **2.** $19=4+3x$. **3.** $7y-11=24$.

4. $69=4b+65$. **5.** $3a+23=104$. **6.** $5\mathrm{V}-13=72$.

7. $89-2n=9n-10$. **8.** $23-\mathrm{W}=2\mathrm{W}+5$.

9. $38+c=2+9c$. **10.** $5d+5=17-3d$.

11. $9x-7=817+x$. **12.** $5z-11=4+2z$.

13. A certain number is multiplied by 7, and 13 is added to the result; if the final number is 69, what was the original number?

14. Think of a number; double it; add 15. If the result is 69, what was the number first thought of?

15. A certain number is multiplied by 3 and 5 is taken from the product; if the result is 46, what was the original number?

16. Find a number such that if 7 is added to three times the number, the sum is 100.

17. The sum of three times x and 7 is 40; what is x?

18. Five times x is 12 greater than 23, find x.

19. The difference between four times a number and 11 is 37; what is the number?

3·6. From the above examples you have probably got the idea that to solve an equation we try to get the terms containing the hidden number on one side of the equation and the terms containing only known numbers on the other side.

Exercise 3 c (Oral)

1. How would you get rid of the term containing the hidden number on the right-hand side of each of the following equations?

(i) $5x = 3x + 8$, (ii) $7a = 36 - 5a$,

(iii) $b = 20 - b$, (iv) $\frac{1}{2}y = 1 - y$.

2. How would you get rid of the term containing a letter on the left-hand side of each of the following equations?

(i) $14 + n = 3n$, (ii) $21 - 3x = 4x$, (iii) $9\frac{1}{2} - 2p = p$.

3. What would you do to the following equations so as to bring all the terms containing an unknown to the left-hand side and all the other terms to the right-hand side?

(i) $5x + 2 = 3x + 6$, (ii) $3y - 7\frac{1}{2} = 2y - 5\frac{1}{2}$,

(iii) $3z - 3 = 2 + 5$, (iv) $4p - 2 = 10 - 2p$,

(v) $2 - q = 13 - 2q$, (vi) $5z - 2 = 2z + 6 + 12$.

4. Is the equation $7a = 3a + 10$ the same as $7a - 2 = 3a + 8$?

5. Is the equation $7a = 3a + 10$ the same as $2 - 3a = 8 - 7a$?

6. Criticise this attempted solution:

$$3d-3=4d-6,$$
$$\therefore\ 3d-4d=-6+3.$$

7. Find, with as little work as possible, whether $x=1$ is a solution of $57x-56=103x-102$.

8. Check the solution $x=2\frac{1}{2}$ for the equation

$$15-5x=7x-10-2x.$$

9. Is the solution to $4a-3=9-2a$, $a=1$, or 2, or 3?

EXERCISE 3 d (i)

Solve the following equations and check your solutions:

1. $6a=3a+9$. **2.** $10-3x=x$. **3.** $5p+3=7+4p$.

4. $5c-5=9-2c$. **5.** $13-\mathsf{K}=24-2\mathsf{K}$.

6. $4\mathsf{M}-3=2\mathsf{M}+10$. **7.** $1\frac{1}{2}+2f=5\frac{1}{2}$.

8. $5a-2=a+5+2a$. **9.** $15+5x-x-3=10x$.

10. $3q+1=1+3q+2q-7$. **11.** $x+4-2x=3x$.

12. $0=3\mathsf{W}-4-5-2\mathsf{W}$. **13.** $\mathsf{N}-1+2\mathsf{N}-3=7-2\mathsf{N}$.

14. $2s+3s+4s=15$. **15.** $m+\frac{1}{2}+m+\frac{1}{4}=1$.

EXERCISE 3 d (ii)

Solve the following equations and check your solutions:

1. $15+s=4s$. **2.** $4\mathsf{N}-14=3\mathsf{N}$. **3.** $6y+3=4y+6$.

4. $33-4\mathsf{W}=4\mathsf{W}-3$. **5.** $4\mathsf{R}+2=6+3\mathsf{R}$.

6. $4\mathsf{P}-3=2\mathsf{P}+6$. **7.** $2\frac{1}{2}+3g=11\frac{1}{2}$.

8. $11-5s+12=4s+5$. **9.** $3x-5+2x+17=7x$.

10. $2w+7+4w+9=30$. **11.** $2y-7-y-3=0$.

12. $15=5t+3t+15$. **13.** $4-8z+6z+8=0$.

14. $4=3+2t-4-t$. **15.** $3s-4s+7s=3-4+7$.

3·7. Example. The equation $\frac{h}{4}=\frac{72}{5}$ arises in finding the height of a tree by comparing the length of its shadow with that of an upright stick. Find the value of h.

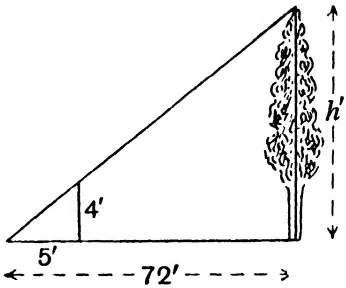


Fig. 3·2

$$\frac{h}{4}=\frac{72}{5}.$$

Multiply each side by 4,

$$\therefore\ h=\frac{72\times 4}{5},$$

$$\therefore\ h=\frac{288}{5},$$

$$\therefore\ h=57\tfrac{3}{5}.$$

Which we check directly, since $\frac{57\frac{3}{5}}{4}=14\frac{2}{5}$, which $=\frac{72}{5}$.

Exercise 3 e

Solve the following equations. Explain every step of your work and check your answers:

1. $\frac{h}{3}=4.$ **2.** $5=\frac{W}{6}.$ **3.** $\frac{2l}{3}=4.$

4. $\frac{1}{2}x=5.$ **5.** $\frac{3d}{4}=7.$ **6.** $\frac{6x}{5}=\frac{2}{3}.$

7. $\frac{h}{5}=1\frac{2}{3}.$ **8.** $\frac{5l}{3}=\frac{28}{5}.$ **9.** $\frac{2a}{3}=1\frac{3}{4}-1\frac{1}{2}.$

10. When the weights in fig. 3·3 balance we can verify by experiment that

$$Wd=wD.$$

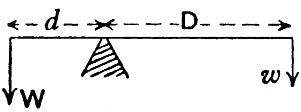


Fig. 3·3

Find what numbers should be put in the spaces:

	W	d	w	D
(i)	25		10	40
(ii)		4	8	10
(iii)	12	3		9
(iv)	24	3	4	
(v)	$2a$	3	$4a$	
(vi)	8	$\frac{b}{2}$		$14b$

3·8. It should be clear that

(i) **if we take the square root (or any other root) of each side of an equation the two sides will again be equal;**

(ii) **if we square (or raise to any other power) each side of an equation the two sides will again be equal.**

3·9. Problem. Fig. 3·4 shows the plan of a square courtyard. The shaded area is 384 sq. yd. Find the length of the edge of the courtyard.

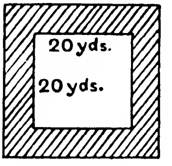


Fig. 3·4

Let the edge of the courtyard be y yd. long.

Then the area of the courtyard is y^2 sq. yd.

And the area of the inner square is 20^2 sq. yd.

$$\therefore\ y^2 - 20^2 = 384,$$

i.e. $$y^2 - 400 = 384.$$

Add 400 to each side.

$$\therefore\ y^2 - 400 + 400 = 384 + 400,$$

i.e. $$y^2 = 784.$$

Hence y is the number whose square is 784.

$$\therefore\ y = 28.$$

Check. If the edge is 28 yd.,

Area of courtyard	$= 28^2$ sq. yd.
	$= 784$ sq. yd.
Area of inner square	$= 400$ sq. yd.
Difference	$= 384$ sq. yd.

Exercise 3 f (Oral)

The equations Nos. 1–12 arise from problems about areas and volumes. Find the unknown numbers.

1. $s^2 = 25.$ **2.** $r^3 = 8.$ **3.** $3a^2 = 12.$

4. $x^2 + 4 = 20.$ **5.** $400 = l^2 - 144.$ **6.** $2d^3 - 1 = 53.$

7. $1 = 9 - p^3.$ **8.** $(2a)^2 = 100.$ **9.** $(2s)^3 = 8.$

10. $\sqrt{x} = 3.$ **11.** $\dfrac{h^2}{4} = 9.$ **12.** $64W^3 = 1.$

EXERCISE 3 g

1. If P is the perimeter of the rectangle in fig. 3·5, we know that

$$P = 2l + 2b.$$

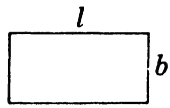


Fig. 3·5

Fill in the spaces:

	P	l	b
(i)		4	3
(ii)	10	2	
(iii)	12		3

2. If A is the area of fig. 3·5, we know that $A = lb$. Fill in the spaces:

	A	l	b
(i)		4	5
(ii)	20	5	
(iii)	24		6

3. The area of fig. 3·6 is $\frac{1}{2}s^2$. If $s = 5$, find the area. If the area $= 50$, find s.

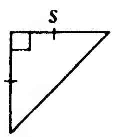


Fig. 3·6

4. If T stands for the total area of faces of a solid cube with edge e and V for the volume, then we can prove that $T = 6e^2$, $V = e^3$.

Fill in the spaces:

	T	V	e
(i)			2
(ii)	54		
(iii)		1000	

5. To make the wire frame shown in fig. 3·7 we require 11 ft. of wire. Find l.

6. What value of s will make the rectangle of fig. 3·8 a square?

7. The extending ladder in fig. 3·9 has a total extended length of 45 ft. Find x.

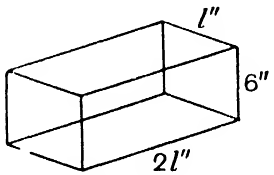


Fig. 3·7

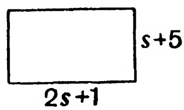


Fig. 3·8

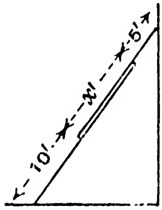


Fig. 3·9

8. Find the distance BQ in fig. 3·10 if AQ is 5 times BQ.

9. Find e if the total outside area of the box in fig. 3·11 is 66 sq. in.

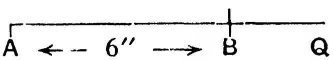


Fig. 3·10

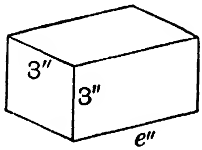


Fig. 3·11

10. A cube has a total surface area of 150 sq. in. Find its volume.

11. When both blades of the penknife in fig. 3·12 are shut, they overlap by $\frac{1}{2}$ in. Find b.

12. By digging the shaded border in fig. 3·13 the gardener doubles the area of the original bed (unshaded). Find d.

13. Find x in fig. 3·14.

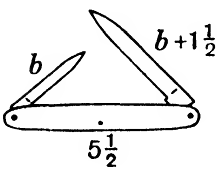


Fig. 3·12

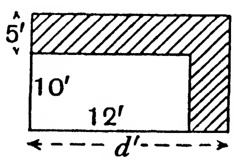


Fig. 3·13

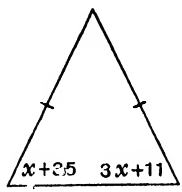


Fig. 3·14

CHAPTER 4

PROBLEMS

4·1. For hundreds of years the chief use of Algebra was the solution of problems.

The chief difficulty for the beginner is in translating the words of the question into the language of symbols.

In the following pages the problems are divided up into types and a few examples are given on each type; in Ex. 4 *g* the problems are mixed.

Note that in each problem the letter assumed stands for a *number* and not for a thing, e.g. in Ex. 4*a* (i), No. 1, x stands for a number (a number of pounds), it does not stand for a sum of money.

EXERCISE 4 a (i)

Solve the following problems. The part of each question which is in brackets should be discussed orally before any work is written.

1. Three men, Mr A, Mr B, Mr C, contribute £432 to start a business; Mr B contributes twice as much as Mr A, and Mr C three times as much as Mr A. How much did each contribute?

[Suppose Mr A gives £x. How much does Mr B give? How much does Mr C give? How much do they all three together give? How much does the question say they give?]

2 Mr A and Mr B have £37 between them: if Mr A has £15 more than Mr B, how much have they each?

[Suppose Mr B has £p. As Mr A has £15 more than Mr B, Mr A must have £ . How much have they between them?]

3. In Rugby football a goal counts 5 points and a try 3 points. One side scored a certain number of goals, but the other side scored twice as many tries, thus winning by 3 points. What were the scores?

[Suppose the first side scored g goals, how many points have they scored? How many tries did the other side score? How many points?]

4. A man made a will leaving £1000 to be divided among his 3 daughters and 4 sons; each daughter was to receive twice as much as each son. What did each daughter and each son receive?

[Suppose each son received £x. How much did 4 sons receive? How much did each daughter receive? How much did 3 daughters receive? How much did 4 sons and 3 daughters receive? But their total receipts were £ .]

5. The length of a table is twice its width. If the table were 2 ft. shorter and 3 ft. wider it would be square. Find its width.

[Suppose the width of the table is w ft. Then the length is ft. If the table were 2 ft. shorter, its length would be ft.]

EXERCISE 4 a (ii)

1. A man bequeathed £140 to his three servants. A was to have twice as much as B; and B was to have three times as much as C. What were their respective shares?

[Suppose that C had £x.]

2. £60 is to be divided between two brothers so that the elder gets £5 more than the younger. How much will they each receive?

[Suppose that the younger receives £y.]

3. A man sells a certain number of magazines at 6*d.* each and twice as many newspapers at 1*d.* each; how many of each has he sold if his receipts amount to 22 shillings?

[Let m be the number of magazines sold.]

4. A man made a will leaving £1700 for his widow and 2 daughters and 3 sons; the widow was to receive £650 and each daughter was to receive twice as much as each son. What did each son receive?

[Let s be the number of pounds each son receives.]

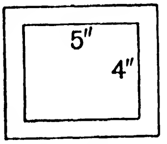


Fig. 4·1

5. The border in fig. 4·1 is everywhere of the same width. The perimeter of the outer rectangle is 26 in. Find the width of the border.

SPEED AND DISTANCE

4·2. Problem. A motor car starts from London to run to Plymouth, 224 miles away; at the same time a second car starts from Plymouth to run to London. The first car goes at 35 miles an hour, the second at 29 miles an hour. How many hours after starting will they meet?

Suppose that they meet t hours after starting.

In 1 hour the first car runs 35 miles.

$\therefore$ in t hours the first car runs $35t$ miles.

Again in 1 hour the second car runs 29 miles.

$\therefore$ in t hours the second car runs $29t$ miles.

$\therefore$ the total distance run by the two cars is $(35t+29t)$ miles

But this total distance must be the distance between London and Plymouth, 224 miles.

$$\therefore (35t+29t) \text{ miles} = 224 \text{ miles},$$

$$\therefore 35t+29t=224,$$

$$\therefore 64t=224.$$

Divide both sides by 64,

$$\therefore t=\frac{224}{64}=\frac{28}{8}=\frac{7}{2}=3\tfrac{1}{2},$$

$\therefore$ the cars meet $3\frac{1}{2}$ hours after starting.

Check. In $3\frac{1}{2}$ hours the first car runs

$$35\times\frac{7}{2} \text{ miles}=\frac{245}{2} \text{ miles}=122\tfrac{1}{2} \text{ miles}.$$

In $3\frac{1}{2}$ hours the second car runs

$$29\times\frac{7}{2} \text{ miles}=\frac{203}{2} \text{ miles}=101\tfrac{1}{2} \text{ miles}.$$

The total distance the cars run is 224 miles, which is the distance between London and Plymouth.

EXERCISE 4 b (i)

1. A train starts from London for Exeter at noon, and another train starts from Exeter for London at the same time; the first train travels at 30 miles an hour, and the second at 57 miles an hour. The distance between London and Exeter is 174 miles. After how many hours will the trains meet?

2. Two motor cars can run one at 30 miles an hour the other at 36 miles an hour. If the faster car sets out to catch the slower when the latter has 16 miles start, when will it catch it up?

3. A thief steals a motor car at 9 a.m. and drives it at 32 miles an hour; the theft is discovered and at 9.30 a.m. the owner sets off in another car at 42 miles an hour; when will he overtake the thief?

EXERCISE 4 b (ii)

1. The distance from London to Crewe is 158 miles. A train leaves Crewe for London at noon travelling at 35 miles an hour; at the same time a train leaves London for Crewe travelling at 44 miles an hour. After how many hours will the trains meet?

2. A liner which can travel at 18 miles an hour receives a wireless message that a steamer 42 miles due west of it has broken down and is travelling due west at only 6 miles an hour. When will the liner overtake the steamer?

3. At 11 a.m. A starts out in a motor car at 28 miles an hour; at noon B rides after him on a motor bicycle at 42 miles an hour. How many miles are they each from the starting-point at x hours after noon? When will B overtake A?

GEOMETRICAL PROBLEMS.

4·3. The following examples assume the properties of angles at a point, parallel straight lines, the isosceles triangle and the angle-sum of a triangle.

Example 1. In fig. 4·2, find x.

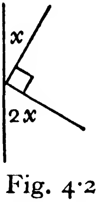


Fig. 4·2

As we have a straight line the three angles make up two right angles.

$$\therefore\ x+90+2x=180.$$

Subtract 90 from both sides, $\therefore\ 3x=90$.

Divide both sides by 3, $\therefore\ x=30$.

Example 2. Find the value of y in fig. 4·3 that would make $\triangle$ ABC isosceles.

[Note that the angles add up to 180°.]

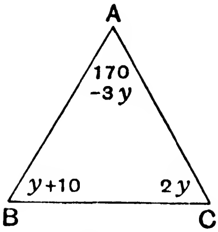


Fig. 4·3

Case I. If AB = AC, $\angle$C = $\angle$B.

$$\therefore\ 2y = y + 10.$$

Subtract y from both sides, $\therefore\ y = 10$.

Check this answer.

Case II. If BA = BC, $\angle$C = $\angle$A.

$$\therefore\ 2y = 170 - 3y.$$

Add $3y$ to both sides, $\therefore\ 5y = 170$.

Divide both sides by 5, $\therefore\ y = 34$.

Check this answer.

Case III. If CA = CB, $\angle$B = $\angle$A.

$$\therefore\ y + 10 = 170 - 3y.$$

Subtract 10 from both sides and add $3y$ to both sides, $\therefore\ 4y = 160$.

Divide both sides by 4, $\therefore\ y = 40$.

Check this answer.

$$\therefore\ y = 10 \text{ or } 34 \text{ or } 40.$$

Example 3. In fig. 4·4, OB = OC; what is $\angle$BOC?

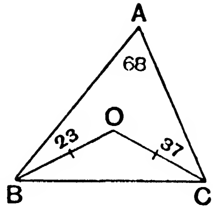


Fig. 4·4

Let $\angle$OBC be x degrees.

[Note that it is simpler in this case to take $x°$ for an angle which is not the angle we are asked for.]

$$\text{OB} = \text{OC},\ \therefore\ \angle\text{OCB} = \angle\text{OBC} = x°.$$

Now the sum of the angles of $\triangle$ ABC is $(68 + 23 + x + 37 + x)$ degrees,

$$\therefore\ 68 + 23 + x + 37 + x = 180,$$

$$\therefore\ 2x + 128 = 180.$$

Subtract 128 from both sides,

$$\therefore\ 2x = 180 - 128,$$

$$\therefore\ 2x = 52.$$

Divide both sides by 2, $\therefore\ x = 26$.

$$\therefore\ \angle\text{BOC} = 180 - 2 \times 26 = 180 - 52 = 128 \text{ degrees}.$$

EXERCISE 4c (i)

Find the values of x:

1. In fig. 4·5.
2. In fig. 4·6.
3. In fig. 4·7.
4. In fig. 4·8.

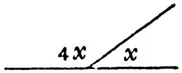


Fig. 4·5

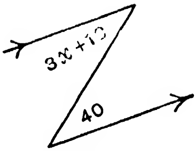


Fig. 4·6

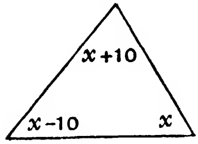


Fig. 4·7

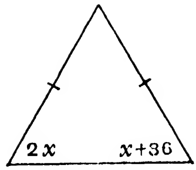


Fig. 4·8

5. In a triangle ABC, $\angle A$ is three times as large as $\angle B$, and $\angle C = 72°$; how many degrees are there in each of the angles A and B?

6. In fig. 4·9, $SA = SB = SC$; find $\angle SBC$.

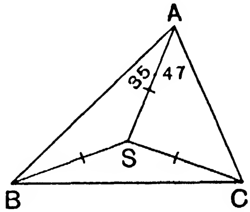


Fig. 4·9

EXERCISE 4c (ii)

Find the value of x:

1. In fig. 4·10.
2. In fig. 4·11.
3. In fig. 4·12.
4. In fig. 4·13.

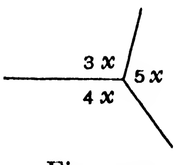


Fig. 4·10

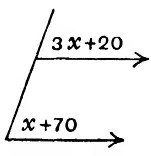


Fig. 4·11

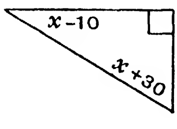


Fig. 4·12

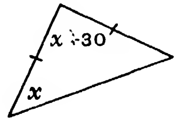


Fig. 4·13

5. A triangle has two of its angles equal and the third angle is 38°; what is each of the equal angles?

6. In fig. 4·14, C is the centre of the circle; calculate z.

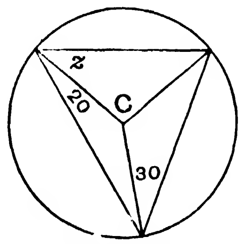


Fig. 4·14

CONSECUTIVE NUMBERS, ETC.

EXERCISE 4d (Oral)

In most of the following questions it will be necessary first to consider numerical cases; in some questions numerical cases are given, in others it is left to the teacher to suggest them when they are needed.

1. The whole numbers (or integers) are 1, 2, 3, 4, ... (i) What is the 12th whole number? (ii) What is the nth whole number?

2. What is the next whole number after 15? If a is a whole number, what is the whole number next after a? next before a?

3. If s is the smallest of three consecutive integers, what are the other two? What is the sum of the three numbers?

4. If g is the greatest of three consecutive whole numbers, what are the three numbers and what is their sum?

5. If m is the middle one of three consecutive integers, what are the three integers? What is their sum?

6. If e is an even number, what is the next greater even number? What is the even number next before e?

7. If d is an odd number, what is the odd number next after d? next before d?

8. If d is an odd number, what is the next greater even number?

9. If x is any integer, what can you say about $2x$? What can you say about $2x+1$?

---

10. If the sum of two numbers is 100, and one of them is n, what is the other?

11. If two men have £40 between them, and if one of them has £p, how much has the other?

12. The height of a tennis net is equal to the sum of the length and the width of a racquet; the height of the net is 36 in. What is the length of a racquet z in. wide?

13. The distance from London to Bristol is 120 miles; when a train has travelled x miles from London towards Bristol, how far is it from Bristol? How far is it from London when it has travelled y miles beyond Bristol?

14. What is the supplement of an angle of (i) 30°, (ii) $x°$?

15. If two of the angles of a triangle are $x°$ and $y°$, what is the third angle?

16. If one of the equal angles of an isosceles triangle is $z°$, what the third angle?

17. If the vertical angle of an isosceles triangle is $v°$, what are each of the equal angles?

Exercise 4 e (i)

1. Find two consecutive numbers which add up to 33.

2. Two consecutive even numbers add up to 70. What are they?

3. The sum of two consecutive odd numbers is 48. Find them.

4. Three consecutive numbers add up to 39. Find the middle one.

5. I have 1*s.* more than my brother. Together we have 18*s.* How much has he?

6. Prove that the sum of two consecutive odd numbers is always divisible by 4.

7. At a concert 100 people were present; x paid 2*s.* and the rest 1*s.* The total receipts were £8. Find x.

Exercise 4 e (ii)

1. Two consecutive numbers add up to 57. What are they?

2. Find two consecutive even numbers which add up to 46.

3. Find two consecutive odd numbers which add up to 36.

4. Three consecutive even numbers add up to 84. Find the middle one.

5. I won a set at tennis by 2 games and the total number of games was 24. How many games did I win?

6. Prove that the sum of three consecutive numbers is always divisible by 3.

7. In a bag there are 10 coins, c of these are sixpences; the rest are pennies. If the value of the money is 2*s.* 6*d.*, what is c?

HINTS FOR SOLVING PROBLEMS.

4·4. I. Pick out from the question what you are asked to find, e.g. "What was the cost?" "Find their ages", ... You will generally find this in the last part of the question.

II. Decide what unknown number you will represent by a letter. If there are two things to be found. e.g. two ages, take a letter for the smaller: this may help you to avoid fractions or negative signs.

III. Write down a complete statement introducing your unknown with its proper unit, e.g. "Let c be the number of pounds in the cost"; or, better still, "Let £c be the cost".

IV. Write down in terms of your unknown each statement in the question. A diagram or a table is often helpful.

V. If you can find two different expressions for any one thing, you can form an equation by equating these two expressions.

N.B. Always remember that the unknown stands for a number only, and not for a number and a unit.

Always check your answer by substitution in the original problem and not in any equation that you have found—you may have made a mistake in writing down the equation.

Exercise 4 f (Oral)

Reword the following statements where necessary:

1. Let c be the cost of the ticket.

2. Suppose he buys n dozen eggs.

3. $3x = 15$, $\therefore$ $x = 5$ years.

4. Let d be the distance.

5. He gains $p - 4$ pence.

6. $\therefore$ the father's age is $2y + 7$.

7. Suppose that W is the weight of the box.

8. Suppose that the number of runs is r.

EXERCISE 4 g (i)

1. r return and $r+4$ single tickets cost 10*s*. A return ticket is 2*s*., a single 1*s*. Find r.

2. If a certain book costs x shillings, what shall I have to pay for 6 copies of the book? How much change shall I receive if I give the bookseller £1? If the change I receive is 5 shillings, what was the price of the book?

3. Fig. 4·15 shows a penknife with open blades. The greatest distance apart of the tips of the blades is 8 in. Find the lengths of the blades.

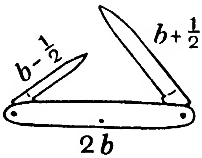


Fig. 4·15

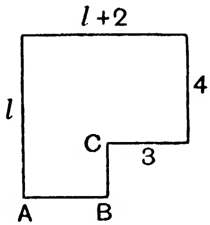


Fig. 4·16

4. In fig. 4·16 lengths are shown in inches. (i) Write down the lengths of AB and BC. (ii) The perimeter of the figure is 32 in. Find l.

5. A man bought 5 lb. of apples at x pence per lb. and 3 lb. of nuts at $2x$ pence per lb.; he paid 5*s*. and received 1*s*. 4*d*. change. What was the price of the apples per lb.?

6. Of four brothers each was four years older than the next younger and the eldest was twice as old as the youngest. What were their ages?

7. Find a in fig. 4·17.

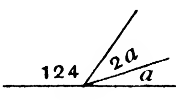


Fig. 4·17

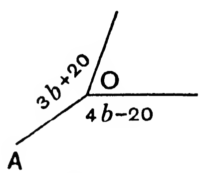


Fig. 4·18

8. Find b in fig. 4·18, if AO produced bisects the acute angle.

9. To roast beef allow 15 minutes for every pound and 15 minutes over. How long will it take to cook pieces weighing 2, 5, n lb.? If a joint takes $2\frac{1}{2}$ hours to roast, how much does it weigh?

10. How far will a man travel in a train journey of 3 hours at $2x$ miles per hour followed by a motor journey of 1 hour at x miles per hour? If the total distance travelled is 161 miles, find the speed of the train.

11. One flock of sheep contains 20 less than twice the number in a second flock. They both together amount to 244 sheep. How many are there in each flock?

12. Fig. 4·19 is a rectangle. Find x.

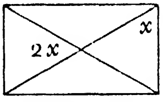


Fig. 4·19

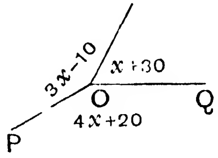


Fig. 4·20

13. In fig. 4·20, prove that POQ must be a straight line.

14. If 10*s.* is divided between two boys A, B, so that A's share is eightpence more than three times B's share, what is B's share?

15. In a triangle ABC the angle A is three times as large as angle B. If B contains 10° more than C, find the angles of the triangle. [Work with angle B as unknown.]

16. (i) Write down the rule for finding the area of a trapezium.

(ii) What must x be for the area of fig. 4·21 to be 30 sq. in.?

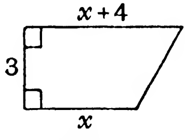


Fig. 4·21

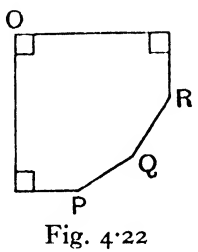


Fig. 4·22

17. Fig. 4·22 shows the plan of a corner shelf. The angles at P, Q, R are equal; call each of them $a°$. Write down the sum of all the angles of the figure.

Also find the sum by dividing the figure into triangles by joining OP, OQ, OR. Hence find a

18. On a coal cart are 18 sacks. n hold 2 cwt. each. The rest hold 1 cwt. each. The total weight is 1 ton. Find n.

19. The expenses of a journey were made up of fares for railway, coach and cab. The railway fare was five times the cab fare, the coach fare was four times the cab fare. The total expense was 30*s.* Find the railway fare.

20. A bag contains a certain number of shillings and twice as many half-crowns; the total value of the money in the bag is £3. How many shillings are there in the bag?

Exercise 4 g (ii)

1. In a cricket match one side scored x runs and $x+67$ runs, the other side scored $2x-3$ runs and $x-50$ runs. If the first side won by 20 runs, what were the scores?

2. Assuming that a return ticket costs twice as much as a single ticket, find an expression for the total cost of 4 singles and 6 returns when a single costs x shillings. What is the cost of a single ticket, if the total cost in the above case is £3. 12*s.*?

3. In fig. 4·23, find x if $AC = 4BC$.

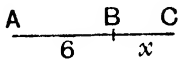


Fig. 4·23

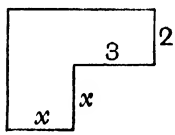


Fig. 4·24

4. Copy fig. 4·24. (i) Mark on your copy the length of each line, in terms of x. (ii) Write down an expression for the perimeter of the figure. (iii) If the perimeter is 15 in., what is x?

5. James starts from Harrow and walks at the rate of 4 miles an hour towards London. At the same instant John starts from London to meet him at the rate of 3 miles an hour. The whole distance being 10 miles, determine when they will meet.

6. (i) Write down the cost in pence of x chickens at 3*s.* 6*d.* each. (ii) Write down their cost in shillings. (iii) If x chickens at 3*s.* 6*d.* each cost x shillings more than £2, find the value of x.

7. Find x in fig. 4·25 if OP bisects the obtuse angle.

8. Find the unmarked angle in fig. 4·26.

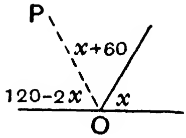


Fig. 4·25

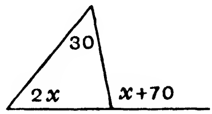


Fig. 4·26

9. How can £1 be shared between A and B so that A receives three shillings more than three times as much as B receives?

10. A garden is covered with lime at the rate of 2 tons per acre on grass and 1 ton per acre on flower-beds. There is three times as much grass as flower-beds and 3 tons in all are used. What area of flower-beds is there?

11. Find x in fig. 4·27. State a fact about the figure and give reasons.

12. Find x in fig. 4·28, and then prove that PQ is perpendicular to QR.

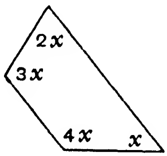


Fig. 4·27

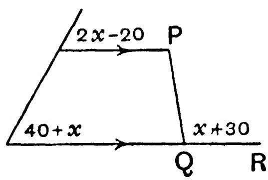


Fig. 4·28

13. One angle of a triangle is 60°; of the other angles the larger is five times the smaller. Find the angles.

14. A batsman scores 150 runs. He hits ten times as many fours as sixes and as many singles as fours. If he scores 38 by his other hits, find how many sixes he hit.

15. In an election one candidate received 826 votes more than the other, and the total number of votes was 4568. How many votes were given for each candidate?

16. A luggage train passes through a station at 20 miles an hour. When it has gone 2 miles further an express going 60 miles an hour dashes through. Where will the collision take place?

17. Show that a triangle whose angles are $\alpha+60$, $\alpha+10$, $\alpha+20$ degrees is right-angled.

18. Find x in fig. 4·29.

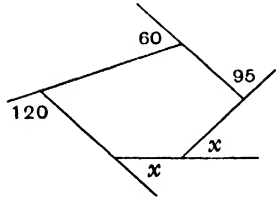


Fig. 4·29

19. A fortress has a garrison of 2600 men, among whom are nine times as many infantry and three times as many artillery soldiers as cavalry. How many of each corps are there?

20. A £5 bag of silver contains only shillings and florins (2*s.* pieces), and there are twice as many florins as there are shillings. Find the number of shillings in the bag.

CHAPTER 5

BRACKETS

5·1. In Arithmetic it is sometimes convenient to enclose an expression in brackets.

Example. To find the area of the four walls of the room shown in fig. 5·1.

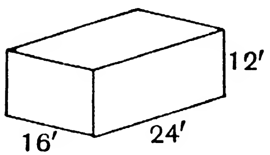


Fig. 5·1

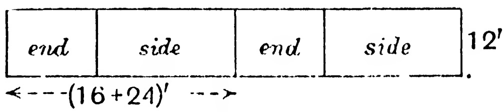


Fig. 5·2

From fig. 5·2 we may write

$$\text{Area} = 2 \times 12 \times (16 + 24) \text{ sq. ft.}$$

$$= 2 \times 12 \times 40 \text{ sq. ft., etc.}$$

Note:

(i) that a bracket binds together the numbers inside it;

(ii) that the contents of a bracket are to be regarded as a single number;

(iii) that, in evaluating an arithmetical expression containing a bracket, the first step is to write the contents of the bracket as a single number.

In Algebra (i) and (ii) above are still true, but (iii) does not apply because we cannot usually combine the contents of a bracket into a single term, e.g.

We can simplify the bracket

in $50 + 2(13 + 7)$ but not in $50 + 2(x + 7)$,

in $180 - (30 + 40)$,, $180 - (y + 40)$.

We must therefore obtain rules for removing brackets from algebraical expressions.

BRACKETS OF THE TYPE a(b + c) AND a(b − c).

5·2. Let us take the following example. Suppose that the diameter of a penny is x in., and of a shilling y in.: let us now put 5 pennies and 5 shillings in a row as in fig. 5·3.

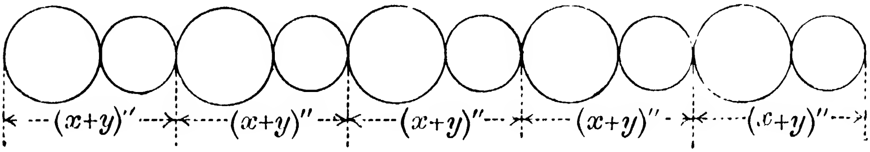


Fig. 5·3

What is the length of each pair? $(x+y)$ in.

What is the total length of the 5 pairs? Five times $(x+y)$ in.: or $5(x+y)$ in.

What is the total length of the pennies? $5x$ in.

shillings? $5y$ in.

whole figure? $(5x+5y)$ in.

$$\therefore\ 5(x+y)=5x+5y.$$

We can generalise this statement by considering fig. 5·4.

The base of the whole rectangle is $(x+y)$ in.

Its area is $a(x+y)$ sq. in.

The areas of the separate parts are ax and ay sq. in.

Hence $a(x+y)=ax+ay$ for all values of the letters.

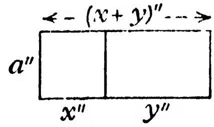


Fig. 5·4

Note particularly that when the brackets are removed from the expression on the left-hand side we must multiply the y by a as well as the x.

Exercise 5a (Oral)

1. Express without brackets:

(i) $4(x+y)$,	(ii) $4(x+1)$,	(iii) $4(1+x)$,
(iv) $a(1+x)$,	(v) $x(1+x)$,	(vi) $a(y+x)$,
(vii) $x(a+y)$,	(viii) $x(x+y)$,	(ix) $2x(a+b)$,
(x) $N(p+q)$,	(xi) $2W(h+2)$,	(xii) $2W(h+\frac{1}{2})$,
(xiii) $\frac{1}{4}(a+4)$,	(xiv) $\frac{2}{3}(6+3x)$,	(xv) $\frac{1}{n}(2n+n^2)$.

2. Express with brackets:

(i) $2y+2x$, (ii) $7a+7b$, (iii) $20+20z$, (iv) $4\mathsf{W}+8$,
(v) x^2+2x, (vi) x^2+xy, (vii) $\frac{1}{2}y+\frac{1}{2}$, (viii) $\frac{1}{2}z+1$.

3. Multiply

(i) $x+7$ by 3, (ii) $a+2$ by 4, (iii) $1+z$ by 10,
(iv) $2+x$ by x, (v) $a+a^2$ by a, (vi) $2+4x$ by $\frac{1}{2}$.

4. Divide

(i) $3a+3b$ by 3, (ii) $2y+4$ by 2, (iii) x^2+x by x,
(iv) $4\mathsf{P}+12\mathsf{Q}$ by 4, (v) $2a^2+2ab$ by $2a$, (vi) $7+14z$ by 7.

Exercise 5b

Some of these may be taken viva voce.

1. Simplify:

(i) $2(a+1)+3(a+2)$, (ii) $3(1+2b)+2(3b+5)$,
(iii) $4(2c+3d)+7(3d+c)$, (iv) $3(\mathsf{W}+\mathsf{P})+2(\mathsf{P}+\mathsf{W})$,
(v) $p(3+q)+q(3+p)$, (vi) $r(s+t)+r(t+s)$,
(vii) $x(1+x)+3(1+x)$, (viii) $w(w+2)+4w(w+3)$,
(ix) $4(x+\frac{1}{2})+6(2x+\frac{1}{3})$, (x) $2a(a+b)+3b(a+b)$.

2. Solve the following equations and check your answers:

(i) $2(x+1)+3=7$, (ii) $12=5(a+2)-3$,
(iii) $4(x+4)=20$, (iv) $2y(y+7)=2(y^2+7)$,
(v) $33=(y+3)\times 2$, (vi) $37=2(3+x)+3(7+x)$.

3. Show that £x, y shillings, z pence $=12(20x+y)+z$ pence.
Express similarly, using brackets, (i) t tons, c cwt., l lb. in pounds, (ii) a yd., b ft., c in. in inches, (iii) m miles, f furlongs, y yd. in yards.

4. Draw diagrams to show that

(i) $x(a+b+c)=xa+xb+xc$, (ii) $x(x+2)=x^2+2x$.

5. Write down the area of fig. 5·5, using (i) one dotted line, (ii) the other dotted line, (iii) both dotted lines. Show that the expressions are identical.

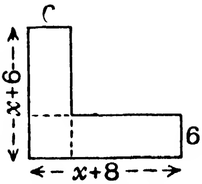


Fig. 5·5

5·3. It is easy to check the result of removing brackets from an expression by giving numerical values to any letter occurring.

Example. Simplify $7(u+v)+4(u-v)$, and test your result by giving numerical values to u and v.

$$\begin{aligned} 7(u+v)+4(u-v) &= 7u+7v+4u-4v \\ &= 11u+3v. \end{aligned}$$

To test the truth of this statement we must give u and v numerical values, but as $u-v$ occurs, the numerical value chosen for u must be greater than the numerical value of v.

Put $\quad u=2, \quad v=1.$

Then $\quad 7(u+v)+4(u-v)=7(3)+4(1)=21+4=25,$

and $\quad 11u+3v=11\times 2+3\times 1=22+3=25,$

which shows that the result is correct for these values of u and v.

Similarly for any other values for u and v.

In fact the R.H.S. is simply another way of writing the L.H.S.

Exercise 5 c

1. Test the following, when $a=2$ and $b=4$:

(i) $4(a+1)=4a+4$, (ii) $7(a+b)=7a+7b$,

(iii) $a(b+1)=ab+a$, (iv) $a(a+b)=a^2+ab$.

2. Find which of the following statements are untrue by testing with $p=1$, $q=2$, and by any other pair of values for p and q:

(i) $q(q+2)=q^2+2$,

(ii) $p^2+4q(p+q)=(p+2q)^2$,

(iii) $p+q+p(p+q)=(p+q)(2p+q)$,

(iv) $(p+q)^2-p^2=q(2p+q)$.

5·7. From the above, and the work we have done before, we should expect, for example, that

$$20+(10+4)=20+10+4, \text{ and generally } d+(b+c)=d+b+c,$$
$$20+(10-4)=20+10-4 \qquad d+(b-c)=d+b-c,$$
$$20-(10+4)=20-10-4 \qquad d-(b+c)=d-b-c,$$
$$20-(10-4)=20-10+4 \qquad d-(b-c)=d-b+c.$$

These generalisations may be summed up in the following rule:

In removing brackets, if the sign before the bracket is +, the + and − signs inside the brackets are unaltered; if the sign before the bracket is −, the + and − signs inside the brackets are changed.

EXERCISE 5f (Oral)

1. Simplify:

(i) $12-(3+1)$, (ii) $12-(3-1)$,
(iii) $12+(3-1)$, (iv) $12+(3+1)$,
(v) $(4+3)-(2+1)$, (vi) $(4-3)-(2-1)$,
(vii) $(a+1)-(a-1)$, (viii) $x-(x-2)$,
(ix) $3p+q-(p+q)$, (x) $p+q-(p-q)$,
(xi) $3(7+4)-(7-4)$, (xii) $a(b-1)-b(a-1)$.

2. Complete the statements:

(i) $4=12-(3+?)$, (ii) $4=12-(12-?)$,
(iii) $2=?-(5-2)$, (iv) $3=4-(?-3)$,
(v) $x=?-(2-x)$, (vi) $P-1=2P-(P+?)$,
(vii) $2s=(r+s)-(?-?)$, (viii) $7=(4+k)+(?-?)$.

3. Fill in the brackets:

(i) $a+b-c=a+(\quad)=b+(\quad)=(\quad)-c$,
(ii) $x-y-z=x-(\quad)=(\quad)-z=(\quad)-y$,
(iii) $p-q+r=r+(\quad)=(p\quad)-q=p-(\quad)$.

4. Prove that when $u=5$, $v=2$, then $(u+v)-(u-v)=2v$ and $(u+v)^2-(u-v)^2=4uv$.

5. Prove that $2(1+b)-2(1-2w)-2(b-2w)=8w$.

5·8. We have seen that

$$a\,(b+c)=ab+ac,$$

and that
$$a\,(b-c)=ab-ac.$$

Hence we may obtain final generalisations of the group of statements in § 5·7.

Thus

$$d+(b+c)=d+b+c \text{ gives } d+a\,(b+c)=d+ab+ac,$$
$$d+(b-c)=d+b-c \quad ,, \quad d+a\,(b-c)=d+ab-ac,$$
$$d-(b+c)=d-b-c \quad ,, \quad d-a\,(b+c)=d-(ab+ac)$$
$$=d-ab-ac,$$
$$d-(b-c)=d-b+c \quad ,, \quad d-a\,(b-c)=d-(ab-ac)$$
$$=d-ab+ac.$$

EXERCISE 5 g (Oral)

1. Fig. 5·7 shows a pole, part of which is painted red and part black. Express the unpainted length (i) with a bracket, (ii) without a bracket.

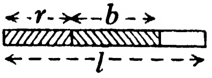


Fig. 5·7

2. Find two expressions for angle C in fig. 5·8, one containing a bracket, the other without a bracket.

Find three expressions for angle A.

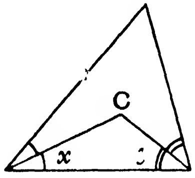


Fig. 5·8

3. Two angles of a triangle are $90-x$ and $90-2x$. Find the third.

4. Simplify:

(i) $12-2\,(3+1)$, (ii) $12-2\,(3-1)$,

(iii) $2x-2\,(x-1)$, (iv) $7-(7-a)$,

(v) $2c+1-2\,(c-1)$, (vi) $3\,(a-b)-2\,(a-b)$,

(vii) $2\,(x-y)-(x-y)$, (viii) $4P-2(P-\frac{1}{2})-2(P+\frac{1}{2})$,

(ix) $x\,(1+x)-x\,(1-x)$, (x) $a(b+c+1)-a(b+c-1)$.

5·9. Sometimes it is convenient to use more than one set of brackets.

Example. Letter post is $1\frac{1}{2}d.$ for the first two ounces and $\frac{1}{2}d.$ for each additional two ounces or fraction thereof.

If a letter weighs z ounces (z being an even number)

$$\text{Postage} = 1\tfrac{1}{2} + \tfrac{1}{2}(z-2) \text{ pence.}$$

If we wish to express the change in pence received out of t shillings, then

$$\begin{aligned} \text{Change} &= 12t - \text{postage} \\ &= 12t - [1\tfrac{1}{2} + \tfrac{1}{2}(z-2)]. \end{aligned}$$

It is sometimes convenient to write $\frac{1}{2}\overline{z-2}$ instead of $\frac{1}{2}(z-2)$.

The line above $\overline{z-2}$ is called a vinculum (Latin for a "bond") and serves to bind together everything underneath, exactly as if these letters and numbers were in a bracket.

Compare the use of a line in a fraction, e.g. in $\dfrac{z+2}{2}$.

When simplifying an expression that has brackets within other brackets, remove the innermost bracket first.

Example. Simplify

$$2[(a+b)-(a-b)] + 3[a\,\overline{b+1} - b\,\overline{a+1}].$$

$$\begin{aligned} \text{The expression} &= 2[a+b-a+b] + 3[ab+a-ab-b] \\ &= 2[2b] + 3[a-b] \\ &= 4b + 3a - 3b \\ &= b + 3a. \end{aligned}$$

EXERCISE 5 h

1. Show that

(i) $3(z-1) + 2(z + 12\,\overline{z+1}) = 29z + 21$,

(ii) $12t - [1\frac{1}{2} + \frac{1}{2}(z-2)] = 12t - \frac{1}{2}z - \frac{1}{2}$,

(iii) $5x - 2[x - 3(2-x)] = 12 - 3x$.

2. Simplify:

(i) $a + 1 + 2\,\overline{a+1} + 3\,\overline{a+1}$, (ii) $a - 1 + 2\,\overline{a-1} - 3\,\overline{a-1}$.

3. Show that the result of expressing p pence, h halfpence, f farthings in shillings is $\frac{1}{12}\left[p+\frac{1}{2}\left(h+\frac{f}{2}\right)\right]$

4. Simplify:

(i) $20-[10-2(p-1)]$, (ii) $1-(1-x)-(2-y)$,

(iii) $13x+4-2[x-(2-x)]$, (iv) $a[4-(4-b)]$,

(v) $3x-[(1+x)-(1+x)]$, (vi) $r\overline{s-2r}-s\overline{s-2r}$,

(vii) $13(p+q)-4[4p+2q-2(q+p)]$.

5. By walking diagonally across the corner I shorten my journey by $\frac{1}{3}[l+b-\sqrt{l^2+b^2}]$ yd. Evaluate this when $l=300$, $b=400$.

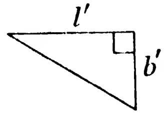


Fig. 5·9

EQUATIONS AND IDENTITIES

5·10. There is a difference between the equalities which we have been writing down in removing brackets and the equations which we solved in Chapter 3.

The statement $5(x+1)=5x+5$ we know to be true when x has any numerical value whatsoever.

On the other hand an equation is true only for certain numerical values of the letters in it. Thus $5(x+1)=15x-5$ is only true when $x=1$; again $x^2+6=5x$ is only true when $x=2$ or when $x=3$.

To **solve** an equation is to find these special values.

These values are called the **roots** of the equation, or its **solutions.**

Such statements as $5(x+y)=5x+5y$, $s-p(r-q)=s-pr+pq$, $3x(x-7)+7(3x-2)=3x^2-14$ are true for all values of all the letters occurring in them. Such a statement is called an **identity** to distinguish it from an equation.

The correct link between the two sides of an identity is $\equiv$, which may be read as "**is identically equal to**". In future we shall use this sign for identities.

An identity is a statement of equality which is true for all values of the letters contained in it. If two expressions are identically equal, either can be transformed into the other by application of the laws of algebra.

EXERCISE 5i

Some of the following statements are identities and some equations. By removing brackets find which are which and find the roots of the equations.

1. $2(x+1)-2(1-x)=4x.$
2. $2(x+1)+2(1-x)=4x.$
3. $2(x+1)+2(1+x)=4(x+1).$
4. $a(b+c)+a=ab+ac+2.$
5. $a(b+c)+d(b+c)=b(a+d)+c(a+d).$
6. $a(a-x)+x(a-x)=a^2-x^2.$
7. $y^2-4(y-1)=y^2+4(y+1)-8y.$
8. $x^3-1=x^2(x-1)+x(x-1)+x-1.$
9. $s(s+1)+2(s+1)=s^2+2s+3.$

EXERCISE 5j

1. Evaluate each of the following when $p=3$, $q=4$, $r=12$:

(i) $p+qr$, (ii) $(p+q)r$, (iii) $r-pq$,

(iv) $(r-p)q$, (v) $r-(p+q)$,

(vi) $(r-p)-(r-q)$, (vii) $pr-p(r-q)$.

2. Write $n=10$, $a=1$, $d=1$ in the expression

$$\frac{n}{2}[2a+\overline{n-1}d]$$

and simplify.

3. Copy each of these statements and complete your copy:

(i) $a+b-c\equiv a+(\quad)\equiv a-(\quad)\equiv b-(\quad)$,

(ii) $\mathsf{P}-2l-2b\equiv \mathsf{P}-(\quad)\equiv \mathsf{P}-2(\quad)$,

(iii) $s^3-4s^2+4s\equiv s(\quad)\equiv s[4-(\quad)]\equiv 4s(\quad)+$

4. Complete the following:

(i) $a\mathsf{B}-c\mathsf{B}\equiv \mathsf{B}(\quad)\equiv \frac{\mathsf{B}}{2}(\quad)\equiv 3\mathsf{B}(\quad)$,

(ii) $y^2z+yz^2\equiv y(\quad)\equiv z(\quad)\equiv yz(\quad)$.

5. Rewrite correctly:

(i) $(2p^2+p)\,2p \equiv 2p^3+2p^2$, (ii) $2h\,(l+b)\times 2 \equiv 4h\,(2l+2b)$,

(iii) $(x-a)\,y-b \equiv x-ay-b$, (iv) $\frac{1}{2}\times 2x\,(2a+4b) \equiv x(a+2b)$.

6. Simplify:

(i) $a\,(b+c)+b\,(c+a)$, (ii) $a\,(b+c)-b\,(c+a)$,

(iii) $2t\,(s-t)-2s\,(t-s)$, (iv) $180-2\,(x+60)$,

(v) $10+\frac{1}{2}\,(2+2k)$, (vi) $T-\frac{1}{n}\,(n+nt)$,

(vii) $2q\,(q^2+1)+q^2\,(1+q)$, (viii) $5y^2\,(2y-3)-5\,(2y-3)$,

(ix) $\frac{1}{m}\,(m-m^2)+2$, (x) $a\,(b-c)-a\,(c-b)$,

(xi) $10\left(\frac{a}{2}+\frac{b}{5}\right)-12\left(\frac{a}{3}-\frac{b}{4}\right)$, (xii) $r\,(s-t)+s\,(t-r)+t\,(r-s)$,

(xiii) $7ax+2a\,(1-x)$, (xiv) $1-3\,(1+x)+2\,(4x-7)$,

(xv) $z\,(1-2z+z^2)-1\,(1+2z+z^2)$, (xvi) $k\left(k+\frac{1}{k}\right)-k\left(k-\frac{1}{k}\right)$,

(xvii) $c^2\,(d-e)+d^2\,(e-c)+e^2\,(c-d)$, (xviii) $\frac{n}{2}\,(2+n)-\frac{n}{2}\,(2-n)$,

(xix) $4-\left[\frac{1}{n}\,(2n-4)\right]$, (xx) $\frac{n}{2}\,[4+4+2\,\overline{n-1}]$.

7. Simplify:

(i) $180-[180-x-(90-x)]$, (ii) $1-[(1-x)+(1-x^2)]$,

(iii) $1-[(1-x)-(1-x^2)]$, (iv) $12t-[1\frac{1}{2}+\frac{1}{2}\,(p-2)]$.

Exercise 5 k

1. Using $(a+b)\,x \equiv ax+bx$, evaluate mentally:

(i) $9\frac{1}{6}\times 12$, (ii) $2\frac{1}{9}\times 36$, (iii) $4\frac{2}{3}\times\frac{3}{4}$, (iv) $1\frac{3}{5}\times\frac{1}{4}+1\frac{2}{5}\times\frac{1}{4}$.

2. If two numbers p and q each have a factor f, is f a factor of

(i) $p+q$, (ii) $p-q$, (iii) $p\times q$, (iv) $p\div q$,

(v) p^2, (vi) p^2+q^2, (vii) p^2-q^2, (viii) $\sqrt{pq}$?

3. Criticise the statement $\dfrac{2a\times 2b}{2a-2b} \equiv \dfrac{ab}{-b}$.

4. If a figure has s sides and its angles all equal, then each angle is $\frac{180}{s}(s-2)$ degrees. Test this for an equilateral triangle and a square.

5. A polygon with n sides has $\frac{1}{2}n(n-3)$ diagonals. Test this for a parallelogram, a 5-gon, a 6-gon and a triangle.

6. If a thermometer reads C° Centigrade and F° Fahrenheit when measuring the same temperature, then $9C = 5(F-32)$.
Test this for the readings of (i) boiling water, (ii) melting ice.

7. Using brackets, write down expressions for the areas of figs. 5·10, 5·11, 5·12. Simplify each expression.

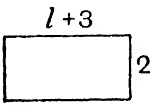


Fig. 5·10

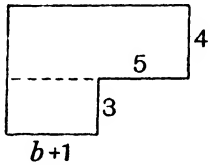


Fig. 5·11

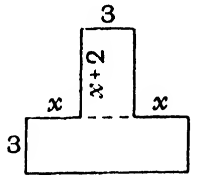


Fig. 5·12

8. If the area of each of figs. 5·10, 5·11, 5·12 is 48 units, find l, b, x.

9. Using brackets, write down expressions for the shaded areas of figs. 5·13, 5·14. Simplify each expression.

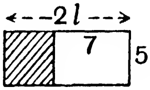


Fig. 5·13

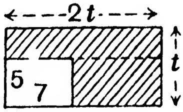


Fig. 5·14

10. If in each of figs. 5·13, 5·14 the shaded area is twice the unshaded area, find l and t.

11. Using brackets, write down expressions for the unmarked angles in figs. 5·15, 5·16, 5·17. Simplify each expression.

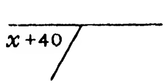


Fig. 5·15

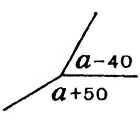


Fig. 5·16

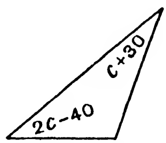


Fig. 5·17

12. If in each of figs. 5·15, 5·16, 5·17 the unmarked angle is 120°, find x, a, c.

CHAPTER 6

EQUATIONS AND PROBLEMS INVOLVING BRACKETS

6·1. Example. Solve the equation $5\,(2x-1)-2\,(4x-2)=2$.

Remove brackets. $\therefore\ 10x-5-8x+4=2,$

$$\therefore\ 2x-1=2,$$

$$\therefore\ 2x=3,$$

$$\therefore\ x=\tfrac{3}{2} \text{ or } 1\tfrac{1}{2}.$$

Check. When $x=1\frac{1}{2}$,

$$\text{L.H.S.}=5\,(3-1)-2\,(6-2)$$

$$=5\times 2-2\times 4=10-8=2,$$

$$\text{R.H.S.}=2.$$

$\therefore$ the solution $x=1\frac{1}{2}$ is correct.

Exercise 6 a (i)

Solve, and check your solutions:

1. $\dfrac{(x-1)}{2}=4.$

2. $\frac{1}{2}\,(45-y)=18.$

3. $\dfrac{2\,(4+z)}{5}=3.$

4. $p-2=\frac{1}{3}\,(p-1).$

5. $3\,(q+5)+2\,(2q-3)=q+9.$

6. $5-(3+l)=1.$

7. $5-(m-3)=4.$

8. $5n-2\,(n+1)=7.$

9. $4f^2+3=3\,(2f^2+1)-2.$

10. $45+\dfrac{r}{2}-\left(90-\dfrac{r}{2}\right)=75.$

11. $32=5\,(s+4)-3\,(s-2).$

12. $0=x^2-x+1-(x^2+x-1)$

13. $2\,(3x-4)+4\,(x+5)=3\,(2x+8).$

14. $3\,(y-4)-2\,(y-2)=5.$

15. $5\,(z-7)+10=2\,(z+7).$

If this step presents difficulties, regard $10x-5-8x+4$ as $10x-4-1-8x+4$, i.e. as $10x-1-8x$.

16. $3\{x+2(x-1)\}=5$.

17. $2(y-3)+5(y+1)=3(y+5)$.

18. $3[(2z-5)-(z-3)]=9$.

19. $3(x-2)-4(2x-3)=2(3x-1)-3(3x+1)+3$.

Exercise 6 a (ii)

Solve, and check your solutions:

1. $\frac{3}{4}(a-1)=9$.

2. $\dfrac{5-h}{2}=1$.

3. $3(d-2)=2(d+3)$.

4. $209=\dfrac{11(25+b)}{2}$.

5. $2(p-1)-(p+1)=0$.

6. $\frac{1}{2}w=\frac{1}{4}(w+4)$.

7. $2(C+3)-3(C-1)=2C+3$.

8. $3(5+x)=2(x-3)+5(1+x)$.

9. $3(5-T)-5(3-T)=0$.

10. $0=9-8(k+1)$.

11. $3-(2p-1)=\frac{1}{2}(p+\frac{1}{2})$.

12. $z^2(18+z)-9(3+2z^2)=0$.

13. $2(2x+7)+5(x-2)=2(3x+5)$.

14. $3(2-3y)-2(5y-1)=3+2y$.

15. $5(z-2)=2(z+3)+z-4$.

16. $8(3x-2)-(7x+4)-15(4-x)=8x$.

17. $3(y-2)=5(y+4)-32$.

18. $6[7(z-1)-13(2-z)]=0$.

19. $18-5(2-x)=7(3-x)-(10-x)-(3-7x)$.

6·2. Example. A father is now 5 times as old as his son. 21 years hence he will be twice as old as his son. What are their ages now?

Let the son's age now be y years.

[Note we take the smaller age as our unknown so as to avoid fractions.]

From the data we can make the following table:

	Now	21 years hence
Son's age	y	$y+21$
Father's age	$5y$	$5y+21$

But in 21 years the father will be twice as old as his son.

$$\therefore\ 5y+21=2(y+21),$$
$$\therefore\ 5y+21=2y+42,$$
$$\therefore\ 3y=21,$$
$$\therefore\ y=7.$$

$\therefore$ The son's age is 7 and the father's 35.

*Check.** In 21 years the son's age will be 28 and the father's 56, and $56=28\times 2$.

Work Ex. 6 *b* (i) or (ii), Nos. 1, 2, 3.

6·3. Example. A purse contains shillings and sixpences. There are 12 coins and their value is 9*s*. How many sixpences are there?

Suppose that there are x sixpences.

Then there are $12-x$ shillings.

Hence we can make the table:

	Sixpences	Shillings	Altogether
No. of coins	x	$12-x$	12
Value in pence	$6x$	$12(12-x)$	$6x+12(12-x)$

But the total value is 9 shillings = 108 pence.

$$\therefore\ 6x+12(12-x)=108,$$
$$\therefore\ x+2(12-x)=18,$$
$$\therefore\ x+24-2x=18,$$
$$\therefore\ 24-x=18,$$
$$\therefore\ 24-18=x,$$
$$\therefore\ x=6.$$

$\therefore$ there are 6 sixpences.

Check. 6 sixpences + 6 shillings = 9 shillings.

$\therefore$ the solution is correct.

[N.B. It would have been easier to have considered the value in sixpences instead of in pence.]

Work Ex. 6 *b* (i) or (ii), Nos. 4, 5, 6.

* Note that we check the actual words of the problem and do not merely check an equation.

6·4. Example. A man rode a distance of 53 miles in 5 hours; for part of the time he rode at 10 miles an hour and for the rest he rode at 12 miles an hour; how many hours did he ride at 10 miles an hour?

Suppose he rode at 10 miles an hour for x hours.

Then he rode at 12 miles an hour for $(5-x)$ hours.

The distance he rode at 10 miles an hour is $10x$ miles

and ,, ,, 12 ,, ,, $12(5-x)$ miles.

$\therefore$ the total distance he rode is $10x+12(5-x)$ miles,

but ,, ,, 53 miles.

$$\therefore\ 10x+12(5-x)=53,$$

$$\therefore\ 10x+60-12x=53,$$

$$\therefore\ 60-53=2x,$$

$$\therefore\ 7=2x,$$

$$\text{or}\ \ 2x=7.$$

$$\therefore\ x=3\tfrac{1}{2}.$$

$\therefore$ he rode at 10 miles an hour for $3\frac{1}{2}$ hours.

Check. In $3\frac{1}{2}$ hours at 10 miles an hour he rode 35 miles.

In $1\frac{1}{2}$,, 12 ,, ,, 18 ,,

Total distance he rode 53 miles.

$\therefore$ the solution is correct.

Work Ex. 6 *b* (i) or (ii), Nos. 7, 8, 9.

Exercise 6 b (i)

1. I am 13 years old and my brother is 3 years old. In how many years' time will his age be half my age?

2. 10 years ago a man was twice as old as his son. Their united ages at present total 89 years. How old is the son now?

3. In 18 years a man's age will be three times his age 14 years ago. What is his present age?

4. Twenty-four coins, consisting of shillings and half-crowns, amount altogether to £1. 19*s*. How many coins are half-crowns?

5. A man bought 39 golf balls, some at 2 shillings each, and some at half-a-crown each. If he paid £4. 9*s*. for the lot, how many of each kind did he get? (Work in sixpences.)

6. In a coal wharf there are 150 bags of coal. Some hold 2 cwt. and the rest 5 cwt. How many of each sort of coal bag are there, if the wharf contains 24 tons of coal?

7. A motor car goes uphill at 20 miles an hour, and downhill and on the level at 40 miles an hour. After 10 hours it has covered 320 miles. How long was it going downhill and on the level?

8. A train starts at noon and travels at 40 miles an hour; how far will it have travelled at x hours after noon? Another train starts from the same place at 2.0 p.m. and travels at 60 miles an hour; how far will it have travelled at x hours after noon? At what time could the second train catch up the first?

9. The Cornish Riviera Express used to leave Paddington for Plymouth at 10.30 a.m. and travelled at 58 miles an hour. When did it meet a train which left Plymouth for Paddington at 11.30 a.m. and travelled at 50 miles an hour? The distance from Paddington to Plymouth is 226 miles.

EXERCISE 6 b (ii)

1. The present ages of a man and his son amount to 55 years. 10 years ago the man was six times as old as his son was then. Find their respective ages.

2. A man, aged 41, has a son 13 years old. In how many years' time will the son be just half his father's age?

3. In 8 years' time a boy's age will be four times his age 7 years ago, how old is he now?

4. A bag contains 65 coins, some of which are half-sovereigns and the rest shillings; if the total amount in the bag is £23. 19*s*., how many coins of each kind are there?

5. I bought 12 railway tickets for 15*s*.; the single tickets cost 1*s*. each and the returns 1*s*. 9*d*. How many singles did I buy?

6. A man bought 10 dozen eggs, some at threepence each, the rest at fourpence each; the whole lot cost £1. 17*s*. 6*d*. How many of each kind did he buy?

7. A man rides 40 miles on a bicycle, partly at the rate of 15 miles an hour, and partly at the rate of 12 miles an hour. For how long must he ride at the former rate in order that he may complete the whole distance in 3 hours?

8. A cyclist leaves London at noon riding 14 miles an hour. Another starts at 1 p.m. and follows him at 20 miles an hour. Write down expressions for the distances, in miles, of the two cyclists from London at t hours after 1 p.m.

Also find an equation to give the time at which the second overtakes the first. What is the time, and how far are the cyclists then from London?

9. An aerodrome receives a wireless message that a Zeppelin has been sighted 20 miles away steering for the coast. After 3 min. an aeroplane travelling $1\frac{1}{2}$ times as quickly as the Zeppelin flies towards it and reaches it in 6 min. Find the speed of the Zeppelin.

Exercise 6 c (i)

1. The lower end of each pole in fig. 6·1 is f ft. below the ground. Find f if one pole is twice as long as the other.

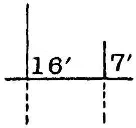


Fig. 6·1

2. £2 is distributed amongst 50 poor persons, each man receives 1*s*. 3*d*. and each woman 9*d*. How many men were there?

3. A has £120 and B has £80; how much must A give B so that B may have twice as much as A?

4. Two travellers, A and B, start on a journey together, A with £105 and B with £48. A spends twice as much as B and at the end of the journey A has four times as much as B. How much has each spent?

5. At a shooting range one pays 2*d*. for a miss, and receives 1*d*. for a hit. A man has 20 shots and has to pay 7*d*. to the keeper of the range. How many hits did he make?

6. In a club the members are divided into three divisions, the second division paying 1*s*. less than the first, and the third 1*s*. less than the second. There are 50 members in the first division, 100 in the second, and 200 in the third, and the total subscriptions are £36. 5*s*. What is the subscription for each division?

7. The vertical angle of an isosceles triangle is v°. What is each base angle? Find v, if each base angle is 30° less than the vertical angle.

8. A sum of £1600 is invested, part of it at 6 %, and the remainder at 7 %. The total income from it is £101. What are the separate investments?

9. A man travels $39\frac{1}{2}$ miles in three hours; for part of the time he walks at 4 miles per hour, for the rest he motors at 15 miles per hour. Find for how long he walked.

10. My hotel bill for 21 days was £17. For the first part of the time the charge was 17*s.* 6*d.* a day, and for the rest it was 15*s.* a day. For how many days was I charged 15*s.* a day?

11. Ten coins lie in a row and each coin is either a shilling or a penny. If the shillings were replaced by pence and the pence by shillings, the money in the row would be increased by 1*s.* 10*d.* Find how many pence are in the row.

12. By cutting off the shaded portion of fig. 6·2 the area of the plot of land was reduced by one-fifth. Find x.

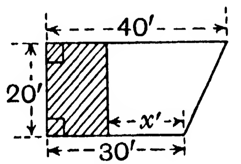


Fig. 6·2

Exercise 6 c (ii)

1. Share 10*s.* between two boys so that one gets half-a-crown more than the other.

2. A has £37 and B has £11; how much must A give B so that A may have twice as much as B?

3. A man bought 2 horses and 5 cows for £135; if each horse cost £8 more than each cow, what was the cost of each?

4. The sum of £3. 6*s.* 6*d.* is made up of thirty coins, which are either florins or half-crowns. How many are there of each?

5. 580 persons were present at a concert for which x of them paid 2*s.* each and the remainder 5*s.* Give the total takings in pounds. What value of x will make the takings amount to £100?

6. The price of eggs having risen $\frac{1}{2}d.$ each, it costs 2*d.* more to buy 20 eggs than it cost to buy 24 eggs before the increase. What was the original price of an egg?

7. A club consists of a certain number of ordinary members who pay £1. 10s. subscription; and a certain number of honorary members who pay 5s. The total number of members (ordinary and honorary) is 66, and the total sum received in subscriptions is £69. How many members are there of each class?

8. Twelve months interest on £400 was £15, but for x months the rate per annum was 3 % and for the remainder of the time it was 4 %. Find x.

9. A man rode a distance of 53 miles in 5 hours; for part of the time he rode at 12 miles an hour and for the rest at 10 miles an hour. For how long did he ride at 12 miles an hour?

10. A screw $\frac{3}{4}$ in. long weighs 1 gm. and a screw 1 in. long weighs 2 gm. If 200 screws, some $\frac{3}{4}$ in. long and the rest 1 in., weigh 240 gm., how many of each kind must there be?

11. In an examination 1930 candidates took additional mathematics, some did two papers, some did three; the total number of papers shown up was 5055. How many candidates took only two papers?

12. In fig. 6·3, DE = DC; find x and all the angles of the figure.

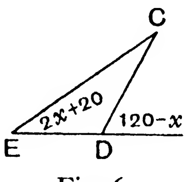


Fig. 6·3

CHAPTER 7

THE CONSTRUCTION AND USE OF GRAPHS

COLUMN GRAPHS

7·1. Here is a set of statistics showing for the various months the average rainfall at a station in the Isle of Wight.

Month	Jan.	Feb.	Mar.	Apr.	May	June
Average rainfall (in.)	2·68	2·05	2·06	1·85	1·72	1·92
Month	**July**	**Aug.**	**Sept.**	**Oct.**	**Nov.**	**Dec.**
Average rainfall (in.)	2·16	2·42	2·24	3·96	3·13	2·29

It is instructive to look at these statistics in the form of a picture or graph. Fig. 7·1 shows them as a column graph.

Average rainfall for various months

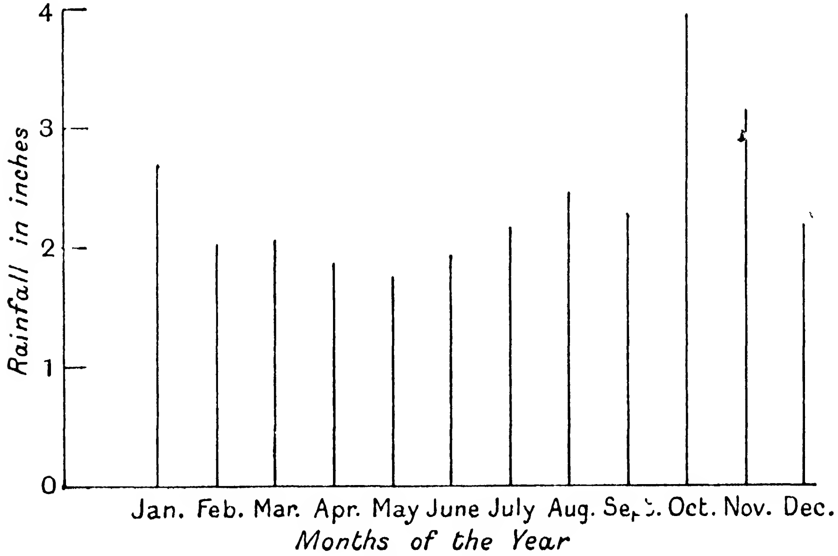


Fig. 7·1

By looking at the graph, find

(i) which is generally the wettest month of the year;

(ii) which is the driest month;

(iii) which months have nearly as much rain as September;

(iv) which is likely to be the worst month for a holiday in (*a*) summer, (*b*) autumn.

7·2. Note the following points about the graph in fig. 7·1 and follow them in all the graphs which you draw:

(i) The graph is given a title.

(ii) The **axes** (i.e. the lines along which measurements are made) are labelled to show what quantities are measured along them.

(iii) The axes are graduated, in one case to show the various months, and in the other to show the number of inches of rainfall.

In order to save time and trouble graphs are usually drawn on paper ruled in inches and tenths (or centimetres and millimetres), but such "squared paper" is not an essential for a graph.

Exercise 7 a

These graphs are to be kept for Ex. 7 b. They may be drawn on plain paper or on squared paper.

1. Draw a column graph showing the average rainfall in the Isle of Wight, i.e. a figure like fig. 7·1. See table on p. 61. Put your uprights $\frac{1}{2}$ in. apart and represent 1 in. of rainfall by a line 1 in long.

2. Draw a graph showing the average maximum temperature at Greenwich for each month.

Month	Jan.	Feb.	Mar.	Apr.	May	June
Temp. in degrees C.	6·4	7·5	10·1	13·9	18·1	21·4
Month	July	Aug.	Sept.	Oct.	Nov.	Dec.
Temp. in degrees C.	23·2	22·6	19·6	14·4	9·4	7·0

Represent 10° C. by 2 in.

Which month has the highest average temperature? Which the lowest?

3. Draw a graph to show the total output of gas in the British Isles from the following figures. Take 1 in. across the page to represent 5 years and graduate from 1900; represent 100 million cubic feet by a line 1 in. long.

Year	1902	1917	1925	1929	1931
Output in 100 million cu. ft.	1·6	2·4	2·9	3·2	3·3

Is the output of gas increasing fairly steadily?

7·3. Sometimes the tops of consecutive uprights are joined by straight lines. This assists the eye to follow the changes in the quantity measured by uprights.

When two sets of measurements are drawn on the same diagram these connecting lines are almost indispensable.

EXERCISE 7 b

1. On your graph of Ex. 7*a*, No. 1, join the tops of consecutive uprights, and then show the rainfall for the various months of 1931; mark the tops of the new uprights by means of a cross.

Month	Jan.	Feb.	Mar.	Apr.	May	June
Rainfall in 1931 in in.	2·05	1·74	1·33	2·44	3·00	2·49
Month	**July**	**Aug.**	**Sept.**	**Oct.**	**Nov.**	**Dec.**
Rainfall in 1931 in in.	3·48	2·95	2·04	0·74	4·14	0·78

In which of the months shown was the rainfall in 1931 near to the average?

In which months was the 1931 rainfall above the average? In which month was it most below the average?

2. On your graph of Ex. 7*a*, No. 2, join the tops of the uprights, and then show the average maximum temperatures in the Isle of Wight, namely:

Month ...	Jan.	Feb.	Mar.	Apr.	May	June
Temp. in degrees C.	7·3	7·3	9·1	11·8	15·5	18·0
Month ...	July	Aug.	Sept.	Oct.	Nov.	Dec.
Temp. in degrees C.	19·6	19·4	17·9	14·1	10·3	8·3

What do you notice about the summer temperatures and the winter temperatures in the Isle of Wight and at Greenwich?

Can you suggest an explanation of this?

3. On the graph of Ex. 7*a*, No. 3, join the tops of the uprights, and then show the number of units of electricity sold. Represent 2 million units by 1 in. draw the upright axis for measuring the units of electricity sold at the right-hand side of your paper.

Year	1900	1910	1921	1930	1931
Millions of units sold	0·1	1·0	3·5	8·7	9·3

What do you notice about the two graphs?

7·4. Instead of drawing the uprights it is usual merely to mark the points where their ends would be as in fig. 7·2. Sometimes the points are marked with fine dots, but in that case it is usual to draw a small circle round each dot so as to draw attention to the dot.

Also the axes are frequently graduated so that the zero line is not shown at all. For example, suppose we have a set of barometer readings taken at various hours of the day, suppose they vary between 746 mm. and 753 mm.; if we were to draw uprights to represent these, the scale would have to be so small that we should not be able to see the rises and falls, which are what we want to see, we therefore take a much larger scale and graduate our bottom line 745 mm.

LOCUS GRAPHS

7·5. Consider the following table, showing a child's weight on its various birthdays:

Age in years	7	8	9	10	11	12
Weight in lb.	48	52	57	62	69	78

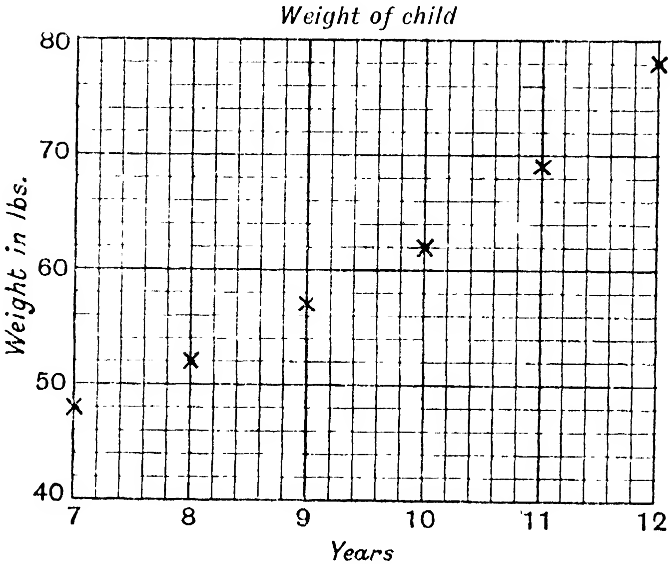


Fig. 7·2

Fig. 7·2 shows these statistics in a graph.

Obviously, if the child's weight had been taken at intermediate times, we should have intermediate points on the graph; in fact, if we could keep a continuous record of its weight, we should get a curve which passed through all the points plotted in fig. 7·2.

As the intermediate points have a meaning, we draw a continuous curve through the given points as in fig. 7·3. From this we can see that the child's weight at $10\frac{1}{2}$ years was probably 65 lb.

This process of getting intermediate values from a graph is called **interpolation.**

A man who has only an ordinary barometer can take frequent readings and mark a lot of points on a graph; he knows that if he took more readings he could get intermediate points on this graph,

so he draws a smooth curve through the points. A barograph would give him a continuous curve, and this would not differ greatly from the smooth curve drawn through the points provided readings are taken frequently.

Whenever you have plotted a set of statistics common sense will generally tell you whether intermediate points have any meaning; if they have, draw a smooth curve through the marked points.

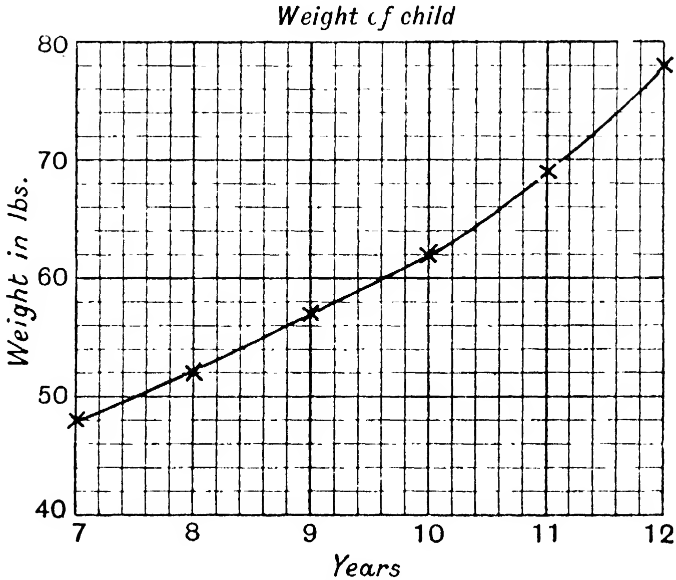


Fig. 7·3

When every intermediate point on a graph has a meaning the graph is called a **locus graph.**

When intermediate points have no meaning, it still may be useful to draw a smooth curve through the plotted points if they lie conveniently, or consecutive points may be joined by straight lines or the points may be left.

EXERCISE 7 c

1. To build an arch of 30 ft. span, a mason was given the heights at various horizontal distances measured from one end of the arch:

Distance in feet	0	5	10	15	20	25	30
Height in feet	0	6·25	10	11·25	10	6·25	0

Make a drawing of the arch on the scale of 1 in. to 5 ft.
Have intermediate points any meaning?
What is the height of the arch at a horizontal distance of 12 ft.? of 24 ft.?

2. Time of High Tide. The following table gives the time of morning high water at London Bridge for certain days in January:

Date	1	3	5	7	9	11	13	15
Time after midnight in hr. and min.	0	1.14	2.34	4.0	5.32	7.21	9.38	12.0

Take 0·4 in. to 1 day across the page and 1 in. to 2 hours along the perpendicular axis. Begin by expressing the minutes as decimals of an hour, to two places.

Draw a smooth curve through the plotted points. What are the times of high water on Jan. 2nd and 10th? Have other intermediate points any meaning?

3. Angle of a polygon. The size of each angle of a regular polygon varies when the number of sides vary.

Number of sides ...	3	4	5	6	8	10
Size of each angle in degrees	60	90	108	120	135	144

Plot a graph and draw a smooth curve.
Have intermediate points any meaning?
Can you tell from the graph the size of the angle of a 7-gon? of a 9-gon?

INDEPENDENT AND DEPENDENT VARIABLES

7·6. From the above exercises it will be clear that

(i) Most graphs are built up from measurements of two quantities.

(ii) One quantity is spaced out from left to right.

(iii) The other quantity is represented by lines drawn upwards or downwards (though in general only the end-points of these lines are shown).

In each of the above graphs the quantity marked from left to right across the page has been the quantity that would be measured first when the information underlying the graph was compiled. These first measurements were selected quite freely—though often they were chosen at equal intervals apart—and only when they had been made was it possible to find the values of the second quantity.

As the quantities may vary through different values it is customary to speak of each as a **variable**. Since the tabulated values of the second quantity depend on such values of the first quantity as we choose to measure, we say that the second quantity depends on the first and it is called the **dependent variable**. By contrast the first quantity which we measure as we please is called the **independent variable.**

It is a convention in drawing graphs that the independent variable is always measured across the page from left to right.

Thus the first thing to be done in preparing a graph from a table of values is to decide which variable is the independent variable.

Notice that we may place our paper so that either the long edge or the short edge runs from left to right.

EXERCISE 7 d (Oral)

Graphs are to be drawn to show the connection between the following: (i) Which quantity should be measured across the page? (ii) Would intermediate points have any meaning?

1. Average weight of men whose heights are 65, 66, 67, ... ins.
2. Number of litres equivalent to 1, 2, 3, ... gallons.
3. Distance fallen by a stone in 1, 2, 3, ... seconds.
4. Date and population of a town.
5. Time of swing of pendulums of various lengths.
6. Days of month and time of sunset.
7. Cost of engines of different horse-power.
8. Resistance to a train travelling at various speeds.
9. Income and income-tax.
10. Height of barometer at different altitudes.
11. Weights of spheres of various radii.
12. Consider the graphs in Ex. 7 *f* (i) and (ii).

CHOICE OF SCALES

7·7. Always choose the scales that will give a fair-sized graph and allow all the data to be used.

If possible 1 in. should represent ., 10, 100, ... or 0·1, 0·01, ... units. The next best scale plan is for 1 in. to represent 2, 20, 200, ... or 0·2, 0·02, ... units, or 5, 50, 500, ... or 0·5, 0·05, ... units.

If 1 in. represents 4 or 40 units it is not so convenient; and a scale in which 1 in. represents 3, 30, ... or 7, 70, ... units should always be avoided, unless all the readings are multiples of 3 or 7 and no intermediate readings are required.

Example. Suppose that you have a sheet of paper 7 in. long (see fig. 7·4) and that your range of readings is from 163 to 274. What scales would you choose?

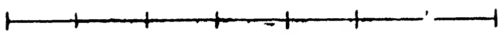


Fig. 7·4

Do this by trial; point to each inch mark and say what graduation you suggest, go along the line and see if all the readings come in.

Try 160, 170, ...; the last line would be 230.

We must choose a smaller scale.

Try 160, 180, ...; the last line would be 300.

We shall waste the last inch of the paper, but we could not avoid that with a simple scale.

EXERCISE 7 e (Oral)

1. With a sheet of paper 7 in. long (see fig. 7·4), what scales would you choose if your readings ranged

(i) from 0 to 12?	(ii) from 5 to 40?
(iii) from 40 to 170?	(iv) from 240 to 820?
(v) from 1861 to 1931?	(vi) from 1 to 4·5?
(vii) from 10 a.m. to 4 p.m.?	(viii) from 0 to 26?
(ix) from 0·1 to 7?	(x) from ·005 to ·012?
(xi) from 5000 to 17,000?	(xii) from 186 to 318?

2. Take a sheet of squared paper on which you will draw the graphs in Ex. 7 *f* (i) and (ii). What scales would be suitable for each of these?

EXERCISE 7 f (i)

1. On the same axes draw graphs to show the populations of Canada and Australia ir millions as given in the following table:

Year	1871	1881	1891	1901	1911	1921
Canada	3·69	4·32	4·83	5·37	7·21	8·79
Australia	1·66	2·25	3·17	3·77	4·45	5·44

What differences do you notice about the changes of population in the two Dominions?

Estimate the population of each in 1887 and in 1914.

2. The error of a range-finder is given in the following table:

Range in yards	1000	2000	4000	6000	8000	10,000
Error in yards	2	8	30	72	130	200

Draw a graph to show the error for various ranges. What are the errors for 3000 and 5000 yards?

3. The following table gives the Canadian fur-catch in millions of pelts and its value in millions of dollars for various years ending in June:

Year	1920	1921	1922	1923	1924	1925
Number	3·6	2·9	4·3	5·0	4·2	3·8
Value	20·4	10·3	17·6	16·6	15·6	15·2
Year	1926	1927	1928	1929	1930	1931
Number	3·7	4·3	3·6	5·2	3·8	4·1
Value	15·0	19·0	18·8	18·8	12·3	11·4

Draw graphs on the same sheet of paper to show these for various years. Take 1 in. to represent a million pelts and 2 in. to represent 10 million dollars.

In which years are the catches greatest and least?

In which two years were the values of the catches greatest?

In which year did the catch go up but the value go down?

4. Show in the same diagram the trajectories or paths of bullets shot from British and German rifles for a range of 1000 yards. In each case the height at each 100 yards is given in feet.

Distance in yd.	0	100	200	300	400	500	600	700	800	900	1000
Height in ft.: British	0	4·9	9·1	12·8	15·6	17·4	17·9	16·7	13·7	8·3	0
German	0	3·8	7·1	9·9	12·2	13·4	13·9	13·0	10·6	6·4	0

Find in what positions a butt 10 ft. high would stop a bullet from each rifle.

5. The following table shows the weight of coal exported from Great Britain and its value for various years:

Year ...	1919	1920	1921	1922	1923	1924
Millions of tons	35·2	24·9	24·7	64·2	74·4	65·5
Millions of £'s	92·3	99·6	42·9	72·5	99·8	78·3

Show these statistics as graphs on the same sheet of paper.

Have intermediate points any meaning?

Are there any noticeable features of the two graphs?

6. The following table gives the displacement in tons and the corresponding draught in feet of a certain ship:

Displacement	3230	3400	3620	3910	4260	4720	5350
Draught	13	$13\frac{1}{2}$	14	$14\frac{1}{2}$	15	$15\frac{1}{2}$	16

Draw a graph showing the displacement against the draught.

If the greatest permissible draught is 16 ft. 3 in., what is the displacement corresponding to this draught?

If when the ship is floating at this greatest draught 800 tons of cargo are unloaded, what will be her draught afterwards?

EXERCISE 7 f (ii)

1. The distance of the horizon for various heights above the sea-level is given in the following table:

Height in feet	0	500	1000	2000	3000	4000	5000	6000
Distance in miles	0	23·8	33·6	47·6	58·25	67·3	75·2	82·4

Draw a graph showing these facts.

What is the distance of the horizon from the top of Snowdon (3560 ft.)?

At what height is the distance of the horizon 40 miles?

2. At a station in the Isle of Wight a vertical rod 10 in. long stood on a horizontal drawing-board and the length of its shadow was measured at various times by the clock as follows:

Time	10.0	10.30	11.0	11.30	12.0	12.30	1.0	1.30	2.0
Length of shadow in inches	12·07	9·80	8·82	8·22	7·95	8·07	8·56	9·38	10·59

Show these on a graph and read off

(i) the length of the shadow at 11.6 and at 1.45;

(ii) the times at which the length of the shadow was 10 in.;

(iii) when the shadow was shortest.

Can you suggest why the shadow was not shortest at noon?

We have already had cases in which we have found an intermediate value, that is called interpolation; but here we have to find a value outside the range of values given, that is called **extrapolation.** Extrapolation is often a dangerous process, often a mere prophecy or guess.

3. Canadian Dominion Government taxation receipts and expenditure per head of the population are shown in the following table:

Year to March 31st	1926	1927	1928	1929	1930	1931
Receipts in dollars	41	42	45	47	44·5	34·2
Expenditure in dollars	38·2	37·5	39·5	38·5	40	42·5

Draw graphs on the same sheet of paper to show the receipts and expenditure in different years, and make a statement about them.

4. The following table shows the progress of the Birmingham water supply:

Year	1895	1900	1905	1910	1915	1920	1925
Revenue in millions of £'s	·181	·234	·271	·304	·340	·494	·705
Daily supply in millions of gallons	16·0	18·2	18·2	19·8	24·1	29·0	25·5

Plot these on the same axes.
Estimate the revenue and daily supply in 1906 and 1914.
What do you notice about the changes in the two quantities?

5. The solubility in water of sodium carbonate at different temperatures is given in the following table:

Temp. in degrees C.	0	10	20	25	30	32·5	35	40	50	60
Solubility	6·6	11·2	17·6	23·0	29·0	34	33·6	33·2	32·2	31·7

Draw a graph, and show how the solubility varies as the temperature rises, noting specially any peculiar feature.
Estimate the solubility at 15° C. and 55° C.

6. The following table shows (in hours and minutes) the time of high tide at Liverpool for the early part of September 1926:

Date, Sept.	1st	3rd	5th	7th	9th	11th
Time	6.18 (a.m.)	8.53 (a.m.)	10.33 (a.m.)	11.40 (a.m.)	12.40 (p.m.)	1.38 (p.m.)

Show this relation by a graph. From the graph find the time of high water on Sept. 4th and Sept. 12th.*

If a captain wished to moor his ship at the landing stage within $1\frac{1}{2}$ hours of high tide, on what days of the period given could he do so at 11 a.m.?

READING OF GRAPHS

Exercise 7 g (Oral)

1. Fig. 7·5 shows the number of boys absent from a day school between January and April.

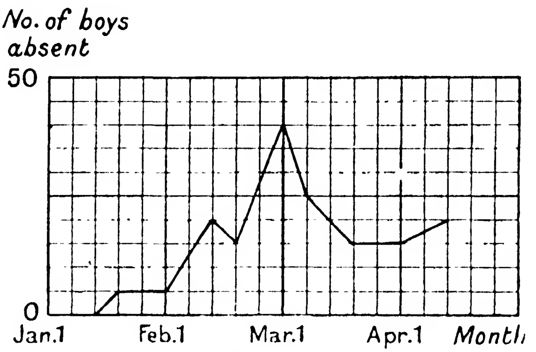


Fig. 7·5

(i) Describe the changes shown by the graph.

(ii) What was the greatest number of absentees in each month?

(iii) There was a heavy fog early in February. About what date was this?

2. Fig. 7·6 shows the total rainfall in inches and the total sunlight in hours from the beginning of March up to the date shown.

* See footnote on p. 72.

(i) Explain, for each graph, what is indicated by the horizontal parts.

(ii) Why does each graph generally go up when the other is horizontal?

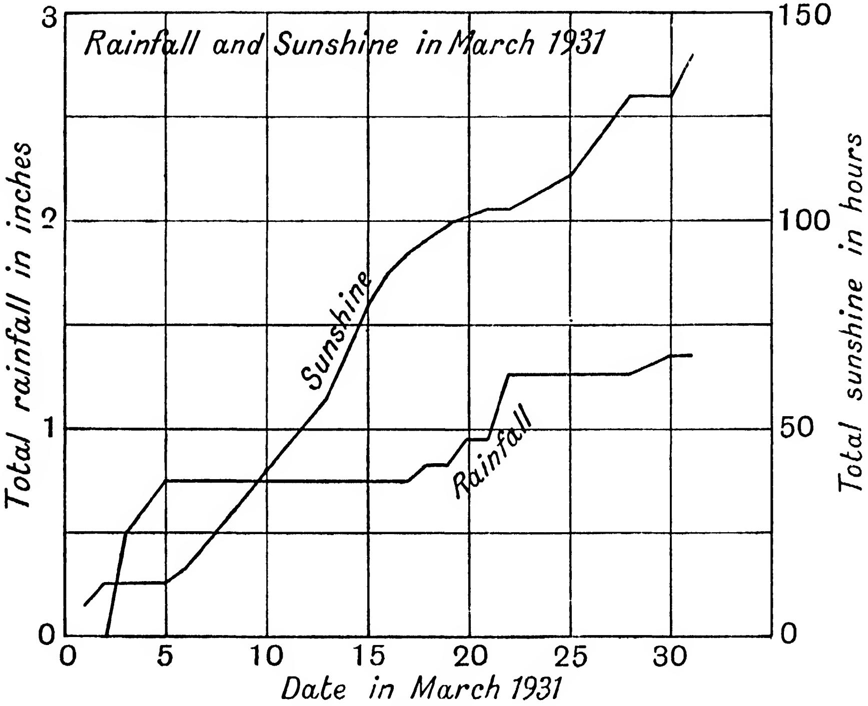


Fig. 7·6

3. Fig. 7·7 shows the numbers of candidates in various years for four different examinations.

(i) Describe the changes in numbers for each examination.

(ii) About what years did the numbers taking C first exceed the numbers taking B and the numbers taking D?

4. Fig. 7·8 shows the national production compared with L.M.S. tonnage and the growth in motor vehicles, standardised on 1927.

Describe for each graph the changes that have taken place in the years 1927 to 1932.

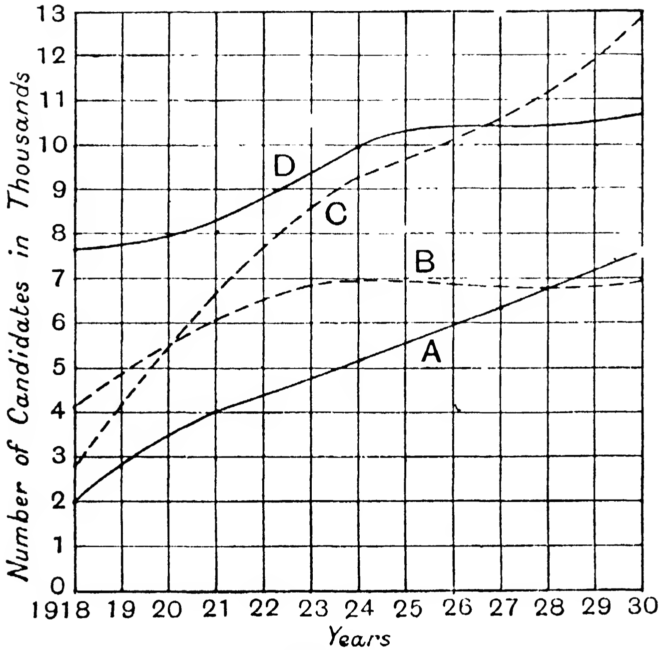
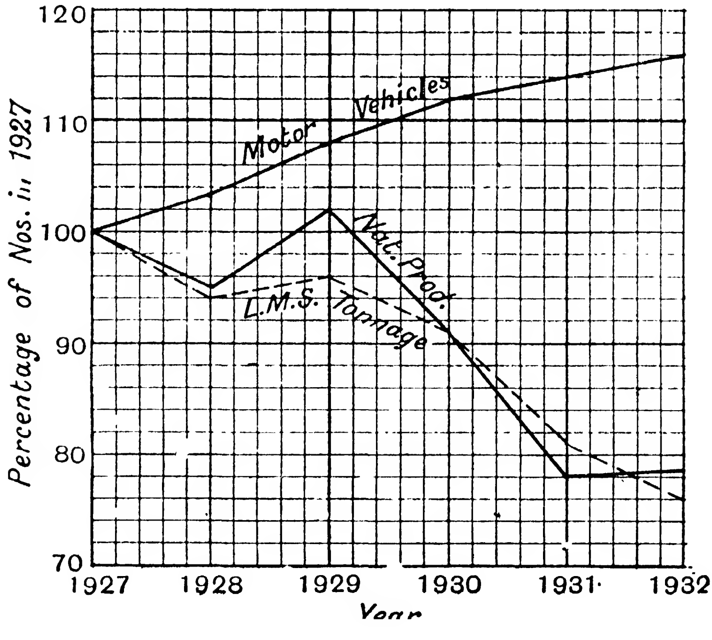

Number of Candidates in Thousands
0
1
2
3
4
5
6
7
8
9
10
11
12
13
D
C
B
A
1918
19
20
21
22
23
24
25
26
27
28
29
30
Years

Fig. 7·7


Percentage of Nos. in 1927
70
80
90
100
110
120
Motor Vehicles
Nat. Prod.
L.M.S. Tonnage
1927
1928
1929
1930
1931
1932

5. The two curves in fig. 7·9 show the average salary earned at various ages by engineers of ability, the one curve applies to those who have had a technical education, and the other to those who have had no such training. What deductions can you draw from a study of the two curves?

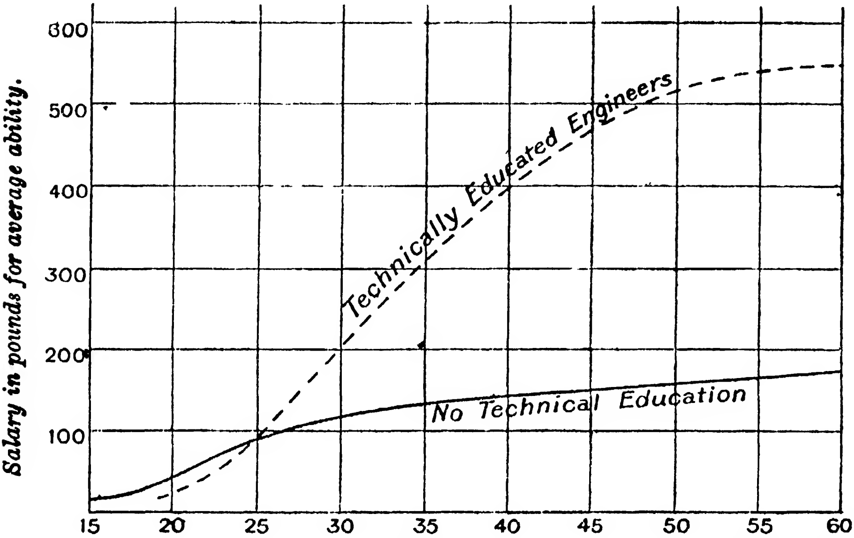


Fig. 7·9

6. Fig. 7·10 illustrates the consumption of electricity and the money invested in the electrical industry since 1900. What conclusions can you draw from comparing the two graphs?

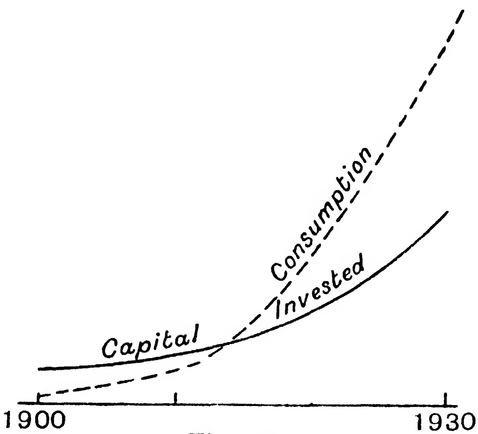


Fig. 7·10

7. Fig. 7·11 shows the horse-power needed to propel a certain destroyer at different speeds, the depth of the water being 30 ft. What remarkable fact is revealed by the graph?

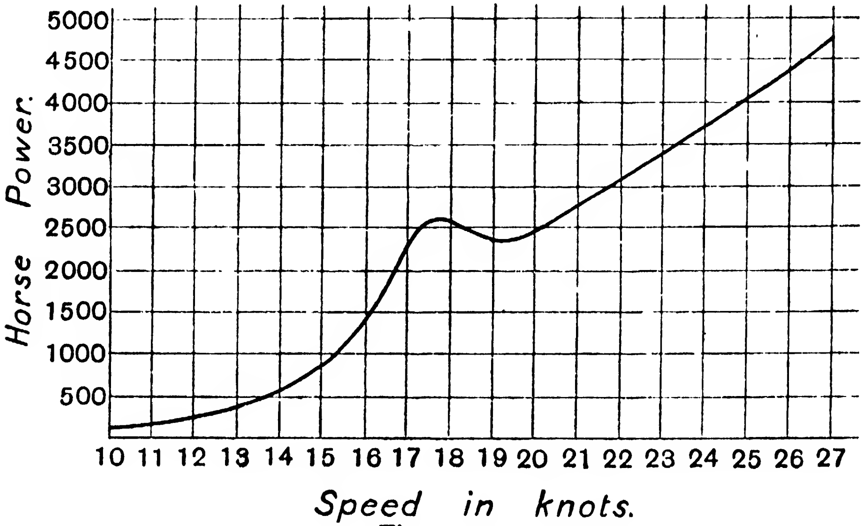


Fig. 7·11

8. Temperature readings are taken during the day for

(i) a point on the surface of the ground;

(ii) a point 2 ft. below the surface;

(iii) a point 4 ft. below the surface.

The three sets of readings are graphed in Fig. 7·12.

Say which set is represented by each graph and account for the shape of each graph.

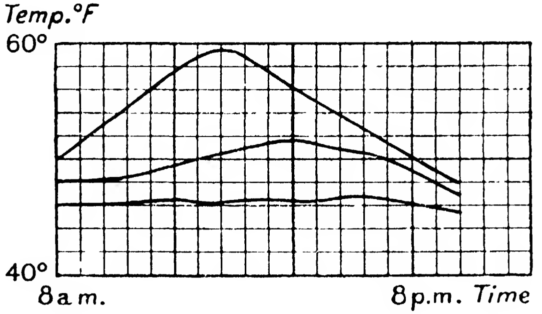


Fig. 7·12

CHAPTER 8

POSITIVE AND NEGATIVE NUMBERS: DIRECTED NUMBER

8·1. So far in this book the signs $+$ and $-$ have been used as connecting links between numbers precisely as in Arithmetic; that is, they have stood for orders to add or to subtract. But we are all accustomed to see single numbers prefixed by these signs. Thus we see temperatures recorded as -4° C. or $+3^\circ$ C. The $+$ and $-$ here do not refer to addition or subtraction, in fact we are using them in a new sense. $+3^\circ$ indicates a point 3° above the zero mark and -4° indicates 4° below it.

8·2. The general idea underlying these new uses of $+$ and $-$ will be brought out by the illustrations below; briefly it may be said that their new use is to indicate direction, up or down, right or left, after or before. Here the $+$ and $-$ are used as adjectives; in Arithmetic and so far in Algebra we have used them as verbs.

For the present we shall use $+$ and $-$ in the old sense as orders to add and subtract and we shall use $\boldsymbol{+}$ and $\boldsymbol{-}$ in this new sense.*

Thus $\boldsymbol{+}5^\circ$ C. will mean 5 degrees measured upwards and $\boldsymbol{-}5^\circ$ C. will indicate 5 degrees measured downwards.

$\boldsymbol{+}5$ we shall call a **positive number** and $\boldsymbol{-}5$ a **negative number,** and either of them may be called a **directed number**.

ILLUSTRATIONS OF THE USE OF POSITIVE AND NEGATIVE NUMBERS

8·3. If dates are reckoned positive from 0 A.D. onwards, the year in which the Great War began is $\boldsymbol{+}1914$, the year 300 B.C. (about which time Euclid wrote his Geometry) would be called $\boldsymbol{-}300$.

If hours of the day are reckoned positive from noon, 3 p.m. would be written $\boldsymbol{+}3$ and 10 a.m. would be written $\boldsymbol{-}2$.

In contour maps, if heights above sea-level are reckoned positive,

* In blackboard work the distinction between $+$, $-$ and $\boldsymbol{+}$, $\boldsymbol{-}$ may be brought out by using coloured chalk for $\boldsymbol{+}$, $\boldsymbol{-}$.

depths below sea-level would be reckoned negative. Thus Snowdon is $+3560$ ft. and the Dead Sea is -1292 ft. above sea-level.

If distances east from any zero point are reckoned positive, distances west are reckoned negative; e.g. if we measure east from Cowes, Beachy Head is $+66$ miles E. and Bournemouth is -24 miles E.

In the same way, if distances north from any zero point are reckoned positive, distances south are reckoned negative; e.g. if we measure north from London, Brighton is -46 miles N., Grimsby is $+140$ miles N.

EXERCISE 8a (Oral)

What meaning (if any) is to be attached to the following?

1. The height of a tree-top above my bedroom window is -6 ft.

2. Scotland beat Ireland by -6 points.

3. The boat went -3 miles upstream.

4. A man was -2 years old at the time of the battle of Waterloo.

5. The Duke of Wellington was -36 years old at the time of the battle of Waterloo.

6. A man's bank balance was -15 pounds.

7. A owes B -6 pounds.

8. The clock is -3 minutes fast.

9. The price of eggs has gone up -1 shilling a dozen.

10. Bristol is -108 miles east of London.

11. Oxford is -42 miles north of Rugby.

12. In one year the population of a town increased by -125.

ADDITION AND SUBTRACTION OF DIRECTED NUMBER

8·4. Example 1. By actually pointing to fig. 8·1, fill up the gaps in the following table, which shows the temperature of a body and its rise of temperature in degrees Centigrade:

Initial temperature	$+10$	$+10$	-10	-10
Rise of temperature	$+20$	-30	$+30$	-20
Final temperature				

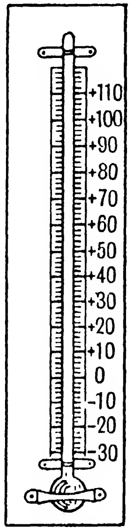


Fig. 8·1

Example 2. By actually pointing to fig. 8·1, fill up the gaps in this table:

Final temperature	$+30$	$+30$	$+10$	-10	-20
Initial temperature	$+10$	-10	$+30$	$+30$	-10
Rise of temperature					

ADDITION

8·5. Fig. 8·2 represents a straight line running north and south and graduated in yards from a point A, $+1$, $+2$, ... to the north and -1, -2, ... to the south.

Let us express in symbols various steps up and down the line.

Consider the expressions $+3 + +2$ and $+3 + -2$:

the $+3$ indicates that we start from the $+3$ mark,

the $+$ indicates that from there we make a further step,

the $+2$ indicates a northward step of 2 yards,

the -2 indicates a southward step of 2 yards.

Make these steps on the diagram, from which we see that

$+3 + +2 \equiv +5$ (i), compare $3+2\equiv 5$;

$+3 + -2 \equiv +1$ (ii), compare $3-2\equiv 1$.

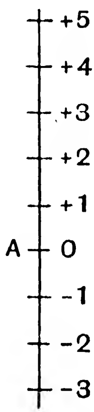


Fig. 8·2

SUBTRACTION

8·6. In Arithmetic, when we wanted to consider $7-5$, we said $5+?=7$, or we said $5+2=7$, therefore $7-5=2$. Using the same idea, we see that

as (i) $+3+\,+2 \equiv +5$, $\therefore$ $+5-\,+2 \equiv +3$ (iii), compare $5-2 \equiv 3$;

as (ii) $+3+\,-2 \equiv +1$, $\therefore$ $+1-\,-2 \equiv +3$ (iv), compare $1+2 \equiv 3$.

But $+1+\,+2 \equiv +3$ (v), compare $1+2 \equiv 3$.

From (ii) and (iii) we conclude that

to add -2 or to subtract $+2$ we come down 2 steps, we move 2 steps in the negative direction, thus, $+\,-2$ and $-\,+2$ are equivalent.

From (iv) and (v) we conclude that

to add $+2$ or to subtract -2 we go up 2 steps, we move 2 steps in the positive direction, thus, $+\,+2$ and $-\,-2$ are equivalent.

8·7. To illustrate that $-(-2)$ is equivalent to $+2$. Consider the following. We regard $+2$ as a step in a forward gear and -2 as a step in reverse gear. Trace the following journeys on fig. 8·2:

A car moves 3 miles N., stops, and then moves forward 2 miles more; it is then 5 miles N. of the starting-point.

Hence $+3+\,+2 \equiv +5$.

A car moves 3 miles N., stops, gets into reverse and moves backwards 2 miles; it is then 1 mile N. of the starting-point.

Hence $+3+\,-2 \equiv +1$.

A car moves 3 miles N., stops, turns round, gets into reverse and moves backwards 2 miles; it is then 5 miles N. of the starting-point.

Hence $+3-\,-2 \equiv +5$, which $= +3+\,+2$.

We may summarise our conclusions as follows:

When in an expression we have $+(-x)$ or $-(+x)$, we may write it as $-x$; when we have $-(-x)$, we may write it as $+x$.

8·8. In Arithmetic we should say $2-5$ is impossible; but, if these are understood to be directed numbers, by reference to fig. 8·2, we see that $+2-\,+5 \equiv -3$.

With this understanding the use of $+$ and $-$ may be dropped,

and + and − used indifferently for symbols of addition and subtraction (verbs) or direction (adjectives). Thus we shall write

$$2-5\equiv-3,$$

$$-13+7+3\equiv-3.$$

Similarly

$$2x-5x\equiv-3x,$$

$$-13s+7s+3s\equiv-3s,$$

$$a^2-10a^2-b^2\equiv-9a^2-b^2.$$

8·9. Draw a line graduated from -10 to $+10$.

Trace on the line the following steps: $4,\ -7,\ 6,\ -2$.

We conclude that $4-7+6-2=1$.

Now trace the same steps but in the following order: $4,\ 6,\ -7,\ -2$.

From this we conclude that $4+6-7-2=1$.

As we might expect the two results are the same.

From this and similar cases we deduce that

In an expression consisting of numbers connected by + and − signs, the numbers may be re-arranged in any order without altering the value of the expression, provided that the signs in front of the numbers are moved with them.

Remember that if no sign stands before a number, you must suppose that there is an unexpressed + sign.

Ex. Express the above statement generally, using a, b, c, d to represent the numbers.

8·10. If we have an expression composed of several like terms, some positive, some negative, it is often best to group the positive terms together and the negative terms together.

Example. $-9p+3p-5p+7p\ [\equiv-9p-5p+3p+7p]$

$$\equiv-14p+10p$$

$$\equiv-4p.$$

The step in brackets may be left out after a little practice.

If positive and negative terms occur in the final result, it is customary to write the positive term first, as this saves writing an extra + sign.

Example. $2a+3b-3a-2b\equiv-a+b$

$$\equiv b-a.$$

Exercise 8 b (Oral)

On the interpretation of directed numbers.

1. Draw a figure like fig. 8·2, by aid of it evaluate the following and write down the corresponding result in arithmetic:

(i) $\mathbf{+}3+\mathbf{+}5$, (ii) $\mathbf{+}3+\mathbf{-}5$, (iii) $\mathbf{-}3+\mathbf{+}2$,
(iv) $\mathbf{-}3+\mathbf{-}2$, (v) $\mathbf{-}3+\mathbf{+}5$, (vi) $\mathbf{-}3+\mathbf{-}5$.

2. Take each of the results in No. 1, consider the corresponding subtractions, and write down their arithmetical equivalents.

[E.g. (vi) $\mathbf{-}3+\mathbf{-}5=\mathbf{-}8$, hence $\mathbf{-}8-\mathbf{-}5=\mathbf{-}3$, arithmetical equivalent $-8+5=-3$.]

Exercise 8c

1. Simplify:

(i) $3-2$, (ii) $2-3$, (iii) $2a-3a$,
(iv) $0+b-2b$, (v) $2+W-3$, (vi) $a+4-5-a$,
(vii) $0-x+2x$, (viii) $0-y-3y$, (ix) $5-c^2-10+c^2$,
(x) $2-2x-3-3x$, (xi) $z-z-1-1$,
(xii) $a-b-b-a$, (xiii) $x-y+z-x+y+z$,
(xiv) $2s^3+s^2-s^3-2s^2$, (xv) $3t+t^2-2t-t^2$.

2. If an explorer in a day advances y miles northward across an ice-floe, while the ice-floe itself drifts $2y$ miles southward, how much nearer is he to the North Pole at the end of the day?

3. If I earn $2a$ shillings and spend $3a$ shillings in a day, how much richer am I at the end of the day? What algebraic identity does this suggest?

4. If my watch loses $3y$ minutes in the day and gains $2y$ minutes in the night, how much does it gain in 24 hours? What identity does this suggest?

5. Fill in the spaces in the following table:

Initial temperature	$+10$	$+15$	$+8$	-2	-2	$-t$
Final temperature	$+13$	$+11$	-10	-7	$+7$	$+2t$
Rise						
Fall						

6. What temperatures should be inserted in the spaces enclosed by brackets?

(i) $+15+(\quad)=+20$, (ii) $+15+(\quad)=+12$, (iii) $+15-(\quad)=+12$, (iv) $+15-(\quad)=+20$,

(v) $-8+(\quad)=-9$, (vi) $-8-(\quad)=-9$, (vii) $-8+(\quad)=+8$, (viii) $-8-(\quad)=+8$.

REMOVAL OF BRACKETS

8·11. In Chapter 5 we developed certain rules for dealing with brackets, but we were restricted because the numerical value of the contents of a bracket had always to be positive; the above work should make it clear that with directed number the same rules still hold but that the restriction to positive values for the brackets is no longer necessary. The rules are so important that we restate them here.

In removing brackets, if the sign before the bracket is +, the + and − signs inside the bracket are unaltered; if the sign before the bracket is −, the + and − signs inside the bracket are changed.

Exercise 8 d

These should first be discussed orally.

1. Simplify:

(i) $a+3-(a+3)$, (ii) $a+3-(a-3)$,

(iii) $a+3-(3-a)$, (iv) $a+3-(3+a)$.

2. Simplify:

(i) $x^2-y-(y-x^2)$, (ii) $-y+x-(x-y)$,

(iii) $-y-x-(y-x)$, (iv) $2(x^2-y^2)-3(x^2+y^2)$.

3. Simplify:

(i) $x(x-2)-2(x^2-x)$, (ii) $a(b-1)-b(a-1)$,

(iii) $4(p+q)-2(2p-2q)$, (iv) $a(1+2x)-x(1+2a)$.

4. Use brackets to express the following; remove the brackets and simplify:

From (i) $W+3$ subtract $2-W$,

(ii) a^2+3c^2 $\quad 3c^2-a^2$,

(iii) $x+y+1$ $\quad x+y+2$,

(iv) 0 $\quad a+b$.

5. Subtract (i) $a+b+c$ from $2a-b+c$,
(ii) $2u-3v$ $3u-2v$,
(iii) x^2+x+1 x^2-x+1.

6. Simplify:

(i) $a(b-c)+b(c-a)+c(a-b)$, (ii) $3(x-4)-4(x-3)$,

(iii) $2(r-2s)-4(r+s)$, (iv) $5(3-x)-7(2+x)$.

7. What should be inside each of the following brackets?

(i) $a-b-c\equiv a-(\quad)\equiv -b-(\quad)\equiv -c-(\quad)$,

(ii) $1-x+x^2\equiv 1-(\quad)\equiv x^2+(\quad)\equiv -x+(\quad)$,

(iii) $ab-a-bc-c\equiv a(\quad)-c(\quad)$,

(iv) $x+3y-6\equiv x+3(\quad)$, (v) $a-4b-4\equiv a-4(\quad)$,

(vi) $u-15v+10\equiv u-5(\quad)$, (vii) $-px+py-p\equiv -p(\quad)$.

MULTIPLICATION OF DIRECTED NUMBERS

8·12. To understand the rule for multiplying two directed numbers we seek some illustration of a product of two factors in which the factors and their product are all three measurable by directed numbers.

From ordinary notions of arithmetic we know that there is a rule

Distance in miles = Speed in miles per hour × Time in hours.

All the quantities in this statement can be represented by directed numbers.

Suppose a man walks steadily during the middle of the day along a road running north and south. Suppose the walker passes a milestone at noon. Choose this milestone as our zero for distances along the road, and distances north of O as positive. Choose noon as our zero for times during which the man has been walking, and times after noon as positive. Then

A distance 8 miles north of O is represented by $+8$,

A time of 2 hours after noon is represented by $+2$.

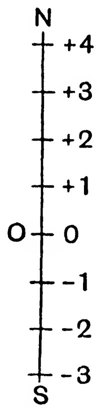


Fig. 8·3

Moreover, we distinguish a speed in a northward direction from a speed in a southward direction by deciding that northward speeds are positive. Thus

A speed of 4 miles per hour northwards is represented by $+4$.

Let us see what consequences follow when we seek to determine the distance of the walker from O after a given time.

(i) Speed $+4$, Time $+2$ (i.e. speed 4 miles an hour northwards, time 2 hours after noon).

Plainly the walker is at $+8$.

This result can only be obtained from the arithmetical rule for finding a distance if we regard

$$+8 \text{ as equivalent to } +4 \times +2.$$

(ii) Speed $+4$, Time -2.

We see that if the walker has travelled northwards for 2 hours at 4 m.p.h., passing O at noon, then 2 hours before noon his distance from O was -8 miles.

We must consider -8 as equivalent to $+4 \times -2$.

(iii) Speed -4, Time $+2$.

We require the distance from O 2 hours after noon, given that the speed is 4 m.p.h. southwards and the walker passed O at noon.

This requires $-8 = -4 \times +2$.

Finally

(iv) Speed -4, Time -2.

We require the distance from O 2 hours before noon, given that the speed is 4 m.p.h. southwards and the walker passed O at noon.

Plainly the distance is $+8$, so we must consider $+8$ as equivalent to -4×-2.

This and further illustrations lead to the conclusion that the only convenient meanings to be given to products of directed numbers, for such fundamental rules as "Distance = Speed × Time" to be obeyed by directed as well as arithmetical numbers, are the following:

$$+a \times +b \equiv +ab,$$
$$+a \times -b \equiv -ab,$$
$$-a \times +b \equiv -ab,$$
$$-a \times -b \equiv +ab.$$

Or in words:

In multiplying together two numbers positive or negative, the numerical value of the product is found by multiplying the numbers in the ordinary way; the sign of the product is + if the factors are of the same sign and − if the factors are of opposite sign.

DIVISION

8·13. Once we have decided that these are to be the laws for multiplying two directed numbers we automatically obtain rules for their division.

Thus each of the above will give two illustrations of a quotient; e.g.

From $$+a \times -b \equiv -ab$$

we deduce $$+a \equiv \frac{-ab}{-b}$$

and $$-b \equiv \frac{-ab}{+a},$$

and we may sum up all the possible results as follows:

$$+a \equiv \frac{+ab}{+b}, \quad -a \equiv \frac{-ab}{+b}, \quad -a \equiv \frac{+ab}{-b}, \quad +a \equiv \frac{-ab}{-b}.$$

Or in words:

In dividing one number by another, whether they are positive or negative, the numerical value of the quotient is found by dividing in the ordinary way; the sign of the quotient is + if the two numbers have the same sign and − if they are the opposite sign.

MULTIPLICATION WHEN ONE OF THE FACTORS IS ZERO

8·14. 0×5 means five times nothing, which clearly is nothing;

$$\therefore \; 0 \times 5 = 0.$$

Again $$5 \times 0 = 0 \times 5 = 0.$$

Also $$-5 \times 0 = -0 = 0.$$

Suppose that there are more than 2 factors; e.g. $5 \times 0 \times 3$. This means that 5 is to be multiplied by 0, giving 0, and the result multiplied by 3, giving 0.

Thus **if one factor of a product is zero, the value of the product is zero.**

It is important to distinguish between "5 multiplied by nothing" and "5 not multiplied by anything".

DIVISION BY ZERO

8·15. It must be clearly understood that $\frac{a}{b}$ or $a \div b$ is quite meaningless when b is zero; in Arithmetic we have never considered division by 0.

Suppose that a man's will directs that his money is to be divided equally between his children that survive him. If he leaves £P and n children survive him, each child will have £$\frac{P}{n}$; but suppose that no children survive him, clearly his will is useless: $\frac{P}{n}$ can have no meaning when $n=0$.

ALGEBRAIC FALLACIES

8·16. Many absurd results *appear* to be proved true by the laws of algebra; in many such cases it will be found that each side of an equation has been divided by something which is zero (generally in some disguised form).

No one would be misled by the following:

Suppose $x=0$, then $\quad 2x=3x;$

divide each side by x, $\quad \therefore\ 2=3.$

But consider this:

If we are asked to solve the equation

$$3x=5x-14.$$

We have

$$3x-21=5x-35,$$

$$\therefore\ 3(x-7)=5(x-7),$$

$$\therefore\ 3=5.$$

In both these cases, the fallacy is due to assuming that it is possible to divide each side of an equation by something which is zero. In the second case the division by zero is not quite so obvious as in the first case.

Exercise 8 e*

Some of these should be taken orally.

1. Simplify:

(i) $(-2)\times(-3)$, (ii) $(-2a)\times(-3)$, (iii) $(-2b)\times(-3b)$,
(iv) $(-5c)^2$, (v) $0\times(-3x)$, (vi) 0^2,
(vii) $y\times(-y)^2$, (viii) $(-a)\times(-a)\times 4$,
(ix) $(-2)\times(-3)\times(-4)$, (x) $(-3)^3$,
(xi) $+3\times(-97)\times 0$, (xii) $2\times(-2)^2\times(-1)$.

2. If $a=-2$, what are the values of a^2, $2a$, $-a$, $a-4$, $a(4-a)$.

3. Find the values of yx, x^2+y^2, $(x+y)(x-y)$, x^2-y^2, $(x-1)(y-1)$; when $x=-3$ and $y=-4$.

4. Make a table showing the values of $x-1$, $x-3$ and $(x-1)(x-3)$ when $x=-3$, -2, -1, 0, $+1$, $+2$, $+3$.

5. Calculate the value of $x(x-1)(x-2)$ when $x=-3$.

6. Simplify:

(i) $\dfrac{12}{-3}$, (ii) $\dfrac{-12}{-3}$, (iii) $\dfrac{-12}{3}$,

(iv) $\dfrac{-12x}{-3}$, (v) $\dfrac{0}{3x}$, (vi) $\dfrac{-0}{3x}$,

(vii) $\dfrac{-2x^2}{-x}$, (viii) $\dfrac{-2\times(-3)}{6}$, (ix) $\dfrac{-3\times 0}{2}$,

(x) $\dfrac{(-2a)^3}{(-a)^3}$, (xi) $\dfrac{-12x}{-24}$, (xii) $\dfrac{3pq}{-3qp}$,

(xiii) $\dfrac{8s^2}{(-2s)^3}$, (xiv) $-\dfrac{1}{n}\times(-n)^2$, (xv) $\dfrac{(-a)(-b)}{ba}$.

7. If $r=-12$, $s=+3$, $t=-2$, calculate

$$\frac{r}{s},\quad \frac{r}{st},\quad \frac{r}{s+t},\quad \frac{r}{t-s},\quad \frac{s^2+t^2}{s+t},\quad \frac{r^2}{st}.$$

* Further practice in evaluation will occur in checking solutions to equations.

8. If $A=b(c+d)$, fill in the spaces:

A	b	c	d
$+20$	-5	-1	
-20	-5	-1	
	$+2$	-2	-2
	$+1$	$+3$	-3
0		$+7$	-4

9. If $W=a(P-Q)$, fill in the spaces:

W	a	P	Q
	$+1$	$+2$	-2
$+12$	$+3$	-2	
-18		-6	-4

Exercise 8 f

1. Write down $(+2)^2$ and $(-2)^2$. If $x^2=4$, what are the *two* possible values for x?

2. Solve, giving all possible solutions, $a^2=9$, $2b^2=50$, $p^3=8$, $q^3=-27$.

3. Has the equation $x^2=-4$ any solutions?

4. Solve, where possible, $144=n^2$, $-125=z^3$, $x^2+1=0$, $y^2=0$.

5. Solve, and check your solutions:

(i) $2(3-x)-(4-x)=1$,

(ii) $2(W+1)-4(1-W)=-8$,

(iii) $d(d-2)-2(d^2-d)=-1$.

6. Why is it impossible for the equation $y^3+y=3$ to have a negative solution?

7. Verify that $x=-2$ is a solution of the equation $x^4-3x^2=4$. Can you write down another solution by inspection?

8. Solve, and check your solutions:

(i) $8-2(3-3x)=2(2x-3)$,

(ii) $3(4s+10)=4(s+5)+30$,

(iii) $-4=p+2(p+1)+3(p+2)$,

(iv) $13+5n-2(8+4n)=9$,

(v) $3(c+2)+2(c+1)+c=0$,

(vi) $3(2x-1)+5(4-3x)=17$,

(vii) $(3y+16)-(y+5)+7(y+2)=(3y+13)-4(y-2)$,

(viii) $x(x-3)+3(x-3)=0$,

(ix) $y(y^2-y+1)+(y^2-y+1)=0$,

(x) $43=3\{12-2(z-1)\}$.

EXERCISE 8 g (i)

Solve the following equations:

1. $7-5(x-2)=5-3(x+3)$. **2.** $15x=3(x-1)-4(1-x)$.

3. $5(3-x)-3(5-x)=0$. **4.** $x-2(x-3)=5x+3(2x+2)$.

5. $3(2-3x)-2(5x-1)-3-2x=0$.

6. $4x-4(x-3)=2-2(x-3)$.

7. $4(x-1)-4x-1=3(2-x)-6-x$.

8. $3\{x+2(x-1)\}=5$.

9. $(13+5x)-2(8+4x)=9$.

10. $10+(x-3)-(x-2)+2(x-1)=11+(x-1)-(x-2)$.

11. $4+(7x-1)-7(x-1)=3+(3x+8)+3(x+8)$.

12. $(3x+16)-(x+5)+7(x+2)=(3x+13)-4(x-2)$.

13. $3[(2x-5)-(x-3)]=9$.

14. $6[17(x-1)-14(2-x)]=0$.

15. $4(2-x)-3(7x+5)+6(3x-1)=-2(x-8)$.

16. $7(x-3)-(7x-3)=5(2-x)-(10-x)-18$.

EXERCISE 8 g (ii)

Solve the following equations:

1. $6(1-x)+4=3(2-x)-4$.
2. $3x-2(12-x)=4(2x-3)+3(x+1)$.
3. $0=3(2-3x)-7(x-3)$.
4. $2(x-1)-3(2-x)+4(1-x)=0$.
5. $2-10(x-2)=3-5(x-3)$.
6. $1=2-(1-y)-2(2-y)$.
7. $5(2.\overline{x+1}+3.\overline{1-x})=0$.
8. $3(2-3x)-2(5-x)=(x+4)-3(2x+1)$.
9. $(9x-6)-3(x-7)=(x+1)+8x$.
10. $1+(5x-15)-(2x-6)=(3x+6)+3(x+2)$.
11. $5x+2-5(x+2)=3(x-5)+3x-5$.
12. $8(3x-2)-(7x+4)-15(4-x)=8x$.
13. $4[3(2-x)-4(x-2)]=12$.
14. $3(x-2)-4(2x-3)=2(3x-1)-3(3x+1)+3$.
15. $3(x-2)-(19-6x)=(5x+61)-3(9x+4)$.
16. $8(x-4)-4(2x-7)=3(10-x)-2(17+x)$.

ADDITION

8·17. In simplifying such an expression as

$$(4-x^2-2x)+(3x^2-2x-1)+(x^2-5+x),$$

which is the sum of the three expressions in the brackets, we may write the expressions in columns as in arithmetic and add by columns:

$$\begin{array}{r} -x^2-2x+4 \\ 3x^2-2x-1 \\ x^2+\ \ x-5 \\ \hline 3x^2-3x-2 \end{array}$$

A more rapid method is to pick out the x^2 terms in the expression as given, namely $-x^2$, $+3x^2$, $+x^2$, and write down their sum repeating the process for the x terms and for the numbers.

Exercise 8 h

Add together:

1. $3x-2y-4,\ x-4y-5,\ 4x-y.$

2. $p+3q-6,\ 3p-q-2,\ 4p-2q-3.$

3. $x^2+x+1,\ 2-x-2x^2,\ 3+x-x^2.$

4. $2a-4b,\ 6b-a-4c,\ 5c-3a-3b.$

5. $2x-3y-5,\ 10-2x+4y,\ 3x+15-y.$

6. $1+x^2,\ 2-3x+5x^2,\ 5x+4.$

7. $2+3y+4y^2,\ 3-3y^2,\ 2(1+y).$

8. $z^3-1,\ z(z-1),\ z+z^2.$

9. What must be added to

(i) $3+4x+5x^2$ to give $6x^2+7x+8$?

(ii) y^2-3y+2 ,, $2y^2-3y-3$?

(iii) $2p^2-3pq+q^2$,, $2p^2-4pq$?

10. Add across in rows, and vertically in columns, and check:

$$\begin{array}{lll} 1+x & x+x^2 & 1-x^2 \\ 3x-x^2 & 2+7x & x-4 \\ x^3-3x & -2x-x^2 & x+1 \end{array}$$

SUBTRACTION

8·18. A subtraction sum may be arranged in column form.

Thus to subtract $2-x+x^2$ from $1+x-x^2$ we write

$$\begin{array}{r} 1+x-x^2 \\ \underline{2-x+x^2} \end{array}$$

and then argue that to subtract 2 is equivalent to adding -2, to subtract $-x$ is equivalent to adding $+x$, i.e. $-(-x)$, and so on.

I.e. **In subtracting we change the sign of each term in the lower line and add.**

Thus here we have

$$-1+2x-2x^2.$$

A more rapid method is to use a bracket and to pick out the terms in the answer, thus

$$1+x-x^2-(2-x+x^2)\equiv -1+2x-2x^2.$$

EXERCISE 8i

1. Copy and complete the following subtraction sums:

$$\begin{array}{r} 2x+5y \\ 3x+4y \\ \hline \end{array} \qquad \begin{array}{r} 7a-2b \\ 2a+\ \ b \\ \hline \end{array} \qquad \begin{array}{r} x+y+3 \\ 2x-y-1 \\ \hline \end{array}$$

2. By using a bracket subtract $1-x+x^2$ from $2-2x+x^2$.

3. Simplify:

$$3a-2b+c-(a-b-2c),$$
$$y^3-2y^2+y+1-(y^3+y^2-y+7).$$

4. From x^2+3x^3 subtract $2+3x+4x^2+2x^3$.

5. Subtract x^2+xy-y^2 from $3x^2+2xy-y^2$.

6. Complete the brackets:

(i) $(1+p+p^2)-(p+p^2+p^3)\equiv(\qquad\qquad)$,

(ii) $(\qquad\qquad)-(a+3x+4)\equiv 2x+2$,

(iii) $(\qquad\qquad)-2a^2-2ab\equiv a^2+ab+b^2$

(iv) $3x^2+6xy+y^2-(\qquad\qquad)\equiv 4x^2+5xy$.

7. What must be subtracted from a^3-3a^2+3a+1 to leave a^3+1?

CHAPTER 9

FRACTIONS WITH VERY SIMPLE DENOMINATORS

9·1. In § 1·4, p. 3, we saw that algebraic fractions behave in the same way as arithmetical fractions, the golden principle being that

The value of a fraction is unaltered by multiplying (or dividing) both its numerator and its denominator by the same number.

Example. Simplify $\dfrac{4a^2b+8ab^2}{12ab^2}$.

Dividing numerator and denominator by $4ab$, we have

$$\frac{4a^2b+8ab^2}{12ab^2} \equiv \frac{a+2b}{3b}.$$

Note that *each* term in the numerator must be divided by $4ab$.

Simplify $\dfrac{\frac{1}{2}bh}{\frac{1}{4}b}$.

Multiply numerator and denominator by 4, then

$$\frac{\frac{1}{2}bh}{\frac{1}{4}b} \equiv \frac{2bh}{b} \equiv 2h.$$

Revise Exercise 1 *c*.

EXERCISE 9a (Oral)

In the earlier questions say what you do to the fraction to simplify it.

Simplify:

1. (i) $\dfrac{2a}{4}$, (ii) $\dfrac{3b}{b}$, (iii) $\dfrac{p^2}{p}$,

(iv) $\dfrac{3x}{9x^2}$, (v) $\dfrac{2z}{\frac{1}{2}}$, (vi) $\dfrac{\frac{1}{2}a}{3}$,

(vii) $\dfrac{s^2}{\frac{1}{2}s}$, (viii) $\dfrac{2pq^2}{pq}$, (ix) $\dfrac{cd^2}{c^2d}$,

(x) $\dfrac{2\pi r^2h}{\pi rh}$, (xi) $\dfrac{\frac{1}{x}}{y}$, (xii) $\dfrac{1}{\frac{p}{q}}$.

2. (i) $\frac{2w+2}{4}$, (ii) $\frac{3(x+1)}{15}$, (iii) $\frac{k^2+7k}{2k}$,

(iv) $\frac{4v-6}{8}$, (v) $\frac{8p-16q}{12}$, (vi) $\frac{3}{6x+6}$,

(vii) $\frac{4}{8(y-2)}$, (viii) $\frac{x}{x-x^2}$, (ix) $\frac{2\pi r^2+2\pi rh}{2\pi r}$

(x) $\frac{\frac{1}{2}(a+b)}{\frac{1}{4}}$, (xi) $\frac{3(r+s)}{\frac{1}{3}(s+r)}$.

3. (i) $\frac{-2R}{4}$, (ii) $\frac{3n}{-6}$, (iii) $\frac{-4z}{-8}$,

(iv) $\frac{-x^2}{2x}$, (v) $\frac{y}{-y}$, (vi) $\frac{-a}{2} \div -4$,

(vii) $\frac{2c-4}{-2}$, (viii) $\frac{-10}{2-2x}$, (ix) $\frac{4u}{4u-u^2}$,

(x) $\frac{-3t}{-3t-3}$, (xi) $\frac{4(c-d)}{6(d-c)}$.

4. (i) $\frac{2}{(2a)^2}$, (ii) $\frac{p^2}{(2p)^2}$, (iii) $\frac{(3x)^2}{x^3}$,

(iv) $\frac{-z^3}{(2z)^3}$, (v) $\frac{(-x)^2}{(-x)^3}$, (vi) $\frac{(qr)^2}{q^2r}$.

Fill in the gaps:

5. $\frac{x}{a} \equiv \frac{\quad}{5a} \equiv \frac{2x}{\quad} \equiv \frac{-xy}{\quad} \equiv \frac{\quad}{a^2} \equiv \frac{xyz}{\quad} \equiv \frac{\quad}{abc}$.

6. $\frac{-1}{q^2} \equiv \frac{-pq}{\quad} \equiv \frac{a}{\quad} \equiv \frac{q}{\quad} \equiv \frac{\quad}{-q^2} \equiv \frac{\quad}{1}$.

7. $\frac{y-1}{2} \equiv \frac{\quad(y-1)}{6} \equiv \frac{5y-5}{\quad} \equiv \frac{ay-a}{\quad} \equiv \frac{\quad}{2a}$

8. $\frac{4p+6q}{5} \equiv \frac{\quad}{15} \equiv \frac{4p^2+6qp}{\quad}$

9. Say which of these are correct:

(i) $\dfrac{2a}{4x^2} \equiv \dfrac{a}{2x^2} \equiv \dfrac{2a^2}{4x^3} \equiv \dfrac{(2a)^2}{(4x^2)^2}$, (ii) $\dfrac{(3q)^2}{(3q)^3} \equiv \dfrac{q^2}{q^3} \equiv \dfrac{1}{q}$,

(iii) $\dfrac{2a+4}{4} \equiv 2a+1$, (iv) $\dfrac{x^2-4x}{4} \equiv \dfrac{x^2-x}{1}$.

10. Solve for x:

(i) $\dfrac{\frac{1}{2}x}{4} = 1$, (ii) $\dfrac{x^2}{4x} = 2$, (iii) $\dfrac{2(x+7)}{6} = 5$.

THE RECIPROCAL OF A NUMBER

9·2. By the **reciprocal** of a number n we mean the number $\dfrac{1}{n}$.

Thus the reciprocal of 3 is $\dfrac{1}{3}$, of x^2 is $\dfrac{1}{x^2}$;

" " $\dfrac{2}{3}$ is $\dfrac{1}{\frac{2}{3}}$, i.e. $1 \times \dfrac{3}{2}$, i.e. $\dfrac{3}{2}$;

" " $\dfrac{p}{q}$ is $\dfrac{1}{\frac{p}{q}}$, i.e. $1 \times \dfrac{q}{p}$, i.e. $\dfrac{q}{p}$.

MULTIPLICATION AND DIVISION OF FRACTIONS

9·3. In Chapter 1 we saw that, from the analogy of arithmetic,

$$\frac{a}{b} \times \frac{c}{d} \equiv \frac{ac}{bd}:$$

and
$$\frac{a}{b} \div \frac{c}{d} \equiv \frac{\frac{a}{b}}{\frac{c}{d}} \equiv \frac{\frac{a}{b} \times bd}{\frac{c}{d} \times bd} \equiv \frac{ad}{cb}.$$

Hence
$$\frac{a}{b} \div \frac{c}{d} \equiv \frac{a}{b} \times \frac{d}{c},$$

or in words:

To divide by a fraction multiply by its reciprocal.

EXERCISE 9 b (Oral)

1. Simplify:

(i) $\frac{1}{2} \times \frac{1}{3}$, (ii) $\frac{1}{a} \times \frac{1}{b}$, (iii) $\frac{2}{5} \times \frac{1}{3}$,

(iv) $\frac{x}{y} \times \frac{1}{z}$, (v) $\frac{3}{4} \times \frac{3}{5}$, (vi) $\frac{p}{q} \times \frac{p}{r}$,

(vii) $\frac{u}{v} \times \frac{v}{w}$, (viii) $\frac{c}{d} \times \frac{d}{c}$, (ix) $\frac{s}{t} \times \frac{s^2}{t}$.

2. What are the reciprocals of

$$\frac{2}{3},\quad 1\frac{1}{4},\quad \frac{a}{b},\quad n,\quad \frac{1}{z},\quad -1?$$

3. Simplify:

(i) $\frac{1}{2} \div \frac{1}{3}$, (ii) $\frac{1}{a} \div \frac{1}{b}$, (iii) $\frac{3}{4} \div \frac{2}{5}$,

(iv) $\frac{p}{q} \div \frac{r}{s}$, (v) $\frac{x}{y} \div \frac{x}{y}$, (vi) $\frac{u}{w} \div \frac{w}{u}$,

(vii) $k \div \frac{1}{k}$, (viii) $\frac{s}{t} \div \frac{1}{t}$, (ix) $\frac{c^2}{d} \div \frac{c}{d}$,

(x) $\frac{12p}{q^2} \div \frac{3pq}{r}$, (xi) $\frac{1}{xy} \div \frac{1}{x^2y}$, (xii) $\dfrac{ad}{\frac{1}{c}}$,

(xiii) $\dfrac{m}{\frac{2}{m}}$, (xiv) $\dfrac{pqr}{\frac{pq}{r}}$, (xv) $\dfrac{x^2}{\frac{1}{x}}$.

4. Simplify:

(i) $\frac{a}{b} \times \frac{c}{d} \times \frac{1}{a}$, (ii) $\frac{1}{x} \times \frac{2}{x} \times \frac{3}{x}$, (iii) $\frac{x}{y} \times \frac{y}{z} \times \frac{z}{x}$,

(iv) $s \times 2s \times \frac{1}{(2s)^2}$, (v) $2b \times b \div \frac{1}{b}$, (vi) $(2c)^2 \times 2c \div c^3$.

5. Solve each of the following for n:

(i) $\frac{n}{2} \times \frac{n}{3} = 6$, (ii) $\frac{n}{2} \times \frac{3}{4} = \frac{3}{4}$,

(iii) $\frac{n}{2} \div \frac{2}{3} = \frac{1}{4}$, (iv) $\frac{2}{n^2} \times \frac{n}{2} = 4$.

6. Simplify:

(i) $4\left(\frac{x}{2} + \frac{y}{4}\right)$, (ii) $12\left(\frac{a}{3} - \frac{b}{4}\right)$,

(iii) $10\left(\frac{x}{15} - \frac{2y}{5}\right)$, (iv) $2z\left(\frac{1}{z} + \frac{1}{4z}\right)$.

ADDITION AND SUBTRACTION OF FRACTIONS

9·4. If two, or more, fractions have the same denominator, we may add or subtract directly.

$$\text{E.g.} \quad \frac{a}{4} + \frac{3a}{4} \equiv \frac{a+3a}{4} \equiv \frac{4a}{4} \equiv a.$$

$$\text{E.g.} \quad \frac{a}{4} - \frac{b}{4} \equiv \frac{a-b}{4}.$$

Note that, **if the numerator of a fraction consists of two or more terms, then these terms must be considered to be in a bracket.**

$$\text{E.g.} \quad \frac{a-b}{4} \text{ is really } \frac{(a-b)}{4} \text{ or } \frac{\overline{a-b}}{4}.$$

9·5. If the denominators of the fractions to be added or subtracted are not the same, the fractions must first be brought to a common denominator, as in arithmetic.

$$\text{E.g.} \quad \frac{x}{4} + \frac{x}{3} - \frac{x}{2} \equiv \frac{3x}{12} + \frac{4x}{12} - \frac{6x}{12} \equiv \frac{3x+4x-6x}{12} \equiv \frac{x}{12},$$

$$\frac{a}{2} + \frac{b}{3} \equiv \frac{3a}{6} + \frac{2b}{6} \equiv \frac{3a+2b}{6}.$$

Note that $\frac{3a+2b}{6}$ is not equal to $\frac{a+2b}{2}$, the denominator has been divided by 3 but only part of the numerator has been so divided.

Example. Express as a fraction with a single denominator $\frac{1}{n^2}-\frac{1}{n}+1$:

$$\frac{1}{n^2}-\frac{1}{n}+1\equiv\frac{1}{n^2}-\frac{n}{n^2}+\frac{n^2}{n^2}\equiv\frac{1-n+n^2}{n^2}\text{ or }\frac{n^2-n+1}{n^2}.$$

Fractions whose numerators contain more than one term present special difficulty. The beginner is advised first of all to enclose these terms in brackets; thus

$$\begin{aligned}\frac{a-2b}{5}-\frac{2a-b}{10}&\equiv\frac{(a-2b)}{5}-\frac{(2a-b)}{10}\\&\equiv\frac{2(a-2b)}{10}-\frac{(2a-b)}{10}\\&\equiv\frac{2(a-2b)-(2a-b)}{10}\\&\equiv\frac{2a-4b-2a+b}{10}\\&\equiv\frac{-3b}{10}.\end{aligned}$$

EXERCISE 9 c (Oral)

Express as fractions with a single denominator:

1. (i) $\frac{a}{2}+\frac{a}{4}$, (ii) $\frac{f}{3}-\frac{f}{4}$, (iii) $\frac{1}{5}x-\frac{1}{10}x$.

2. (i) $\frac{1}{d}+\frac{2}{d}$, (ii) $\frac{7}{z}-\frac{8}{z}$, (iii) $\frac{a}{z}-\frac{b}{z}$.

3. (i) $\frac{1}{m}+\frac{1}{n}$, (ii) $\frac{1}{x}-\frac{1}{y}$, (iii) $p-\frac{1}{p}$.

4. (i) $1+\frac{2}{c}+\frac{3}{c^2}$, (ii) $u+4+\frac{4}{u}$, (iii) $\frac{a}{b}+\frac{b}{a}+2$.

5. (i) $\frac{p^2}{4}-\frac{p^2}{8}-\frac{p^2}{12}$, (ii) $z-\frac{1}{z^2}$, (iii) $\frac{4}{x^3}-\frac{1}{2}$.

6. (i) $\frac{x}{2}-\frac{x-1}{4}$, (ii) $2-\frac{y-1}{2}$, (iii) $\frac{z}{2}-\frac{z+2}{3}$.

7. (i) $a-\frac{a+1}{3}$, (ii) $1-\frac{b+1}{4}$, (iii) $1-\frac{2(c-1)}{3}$.

EXERCISE 9d (i)

Express as fractions with a single denominator:

1. (i) $\frac{2a}{3}-\frac{a}{5}$, (ii) $b-\frac{b}{2}+\frac{2b}{3}$, (iii) $\frac{2}{c}-\frac{3}{2c}$.

2. (i) $\frac{1}{3x}-\frac{1}{3y}$, (ii) $4q+\frac{4}{q}$, (iii) $\frac{x}{a}+\frac{y}{b}$.

3. (i) $\frac{1}{x^2}+\frac{1}{x^3}$, (ii) $a+\frac{a}{b}+\frac{a^2}{b^2}$, (iii) $1-\left(\frac{2}{x}\right)^2-\left(\frac{2}{x}\right)^3$.

4. (i) $\frac{x-1}{2}-\frac{2x+3}{4}$, (ii) $\frac{2y-3}{3}+\frac{y-2}{2}$, (iii) $1+\frac{z-1}{2}$.

5. (i) $\frac{2x-5}{6}-\frac{3x-7}{9}$, (ii) $\frac{y-z}{4}-\frac{z-y}{4}$. (iii) $\frac{5y+2}{14}-\frac{2y+3}{21}$.

6. $\frac{a-2b}{5}-\frac{b-2a}{10}+\frac{2(a+b)}{15}$.

7. $\frac{x-3}{2}-\frac{3(2-x)}{4}+\frac{1}{6}$.

8. $\frac{a+2b}{2}-\frac{3b+c}{3}+\frac{c-a}{8}$.

9. $\frac{b-c}{2}+\frac{c-a}{3}+\frac{a-b}{6}$.

10. $\frac{2}{3}(x-y)-\frac{3}{5}(x+y)$.

11. $\frac{1}{2}(a+b)-\frac{1}{3}(2a-4b)$.

12. $\frac{3x+5}{9}-\frac{5x+3}{2}+\frac{9x-7}{6}$.

13. $\frac{3(x-2)}{12}+\frac{4x-3}{4}-\frac{2x-5}{8}$.

Simplify:

14. $12\left(\frac{x-1}{3}+\frac{2x-3}{4}\right)$.

15. $18\left(\frac{a-2b}{6}-\frac{2a+b}{9}\right)$.

16. $20\left(\frac{x-y}{10}-\frac{x+y}{2}+\frac{3x-2y}{5}\right)$.

17. $16\left(\frac{2a-3b}{8}-\frac{3a-2b}{2}\right)$.

Multiply

18. $\frac{3p-2q}{7}-\frac{2q-p}{21}$ by 21.

19. $\frac{1-2a}{4}-\frac{a-2}{6}+\frac{1}{12}$ by 12.

EXERCISE 9d (ii)

Express as fractions with a single denominator:

1. (i) $\frac{5a}{6}-\frac{4a}{5}$, (ii) $\frac{3b}{4}-\frac{4b}{5}+b$, (iii) $\frac{3}{5c}-\frac{1}{2\frac{1}{2}c}$.

2. (i) $\frac{1}{2a}-\frac{1}{4b}$, (ii) $\frac{W}{2}-\frac{2}{W}$, (iii) $\frac{p}{q}-\frac{q}{p}$.

3. (i) $\frac{1}{a^2}-\frac{1}{ba}$, (ii) $\frac{1}{cd}-\frac{1}{c}-\frac{1}{d}$, (iii) $\left(\frac{3}{y}\right)^2-2\left(\frac{3}{y}\right)+3$.

4. (i) $\frac{3x-2}{5}-\frac{5x-3}{2}$, (ii) $\frac{2y-4}{3}+\frac{3y+5}{4}$, (iii) $2-\frac{2+z}{4}$.

5. (i) $\frac{3x-5}{14}-\frac{x-4}{21}$, (ii) $\frac{y-z}{3}+\frac{z-y}{3}$, (iii) $\frac{2y-5}{9}-\frac{3y+7}{12}$.

6. $\frac{3x-7}{4}-\frac{4(x-2)}{5}-\frac{3-2x}{6}$.

7. $\frac{2x+1}{2}+\frac{3x-2}{5}-\frac{1}{10}$.

8. $\frac{5a+3b}{10}-\frac{2(a+2b)}{15}+\frac{4a-b}{5}$.

9. $\frac{2a-3b}{6}-\frac{3a-4b}{8}+\frac{a}{12}$.

10. $\frac{1}{4}(2x+5)-\frac{5}{6}(3-x)$.

11. $\frac{3}{4}(2x-y+3)-\frac{4}{5}(3x-2y+1)$.

12. $\frac{2y-5}{7}-\frac{3y-4}{4}+\frac{9y-4}{14}$.

13. $\frac{2(x-7)}{8}-\frac{2x-3}{4}+\frac{8-5x}{6}$.

Simplify:

14. $14\left(\frac{x-3}{2}+\frac{2x-5}{7}\right)$.

15. $10\left(\frac{3x-2y}{2}-\frac{6x-5y}{5}\right)$.

16. $12\left(\frac{x+y}{4}-\frac{x-y}{3}+\frac{x}{6}\right)$.

17. $36\left(\frac{7-x}{6}-\frac{2x+5}{9}-\frac{11}{18}\right)$.

Multiply

18. $\frac{x-4}{7}-\frac{x-3}{3}+1$ by 21.

19. $\frac{3\overline{x+3}}{8}-\frac{x-4}{4}$ by 16.

CHAPTER 10

EQUATIONS AND PROBLEMS INVOLVING FRACTIONS

10·1. Example 1. Solve $\frac{x-2}{3}-\frac{12-x}{2}=\frac{5x-36}{4}-2.$

Write the fractions with the numerators in brackets,

$$\frac{(x-2)}{3}-\frac{(12-x)}{2}=\frac{(5x-36)}{4}-2.$$

Multiply each side by 12,

$$\therefore\ 12\times\frac{(x-2)}{3}-12\times\frac{(12-x)}{2}=12\times\frac{(5x-36)}{4}-24,$$

$$\therefore\ 4(x-2)-6(12-x)=3(5x-36)-24,$$

$$\therefore\ 4x-8-72+6x=15x-108-24,$$

$$\therefore\ 10x-80=15x-132,$$

$$\therefore\ 10x-15x=-132+80,$$

$$\therefore\ -5x=-52,$$

$$\therefore\ x=\frac{52}{5}=10\tfrac{2}{5}.$$

Check. When $x=10\frac{2}{5}$,

$$\text{L.H.S.}=\frac{10\frac{2}{5}-2}{3}-\frac{12-10\frac{2}{5}}{2}=\frac{8\frac{2}{5}}{3}-\frac{1\frac{3}{5}}{2}=\frac{42}{3\times5}-\frac{8}{2\times5}=\frac{14}{5}-\frac{4}{5}=\frac{10}{5}=2,$$

$$\text{R.H.S.}=\frac{5\times10\frac{2}{5}-36}{4}-2=\frac{52-36}{4}-2=\frac{16}{4}-2=4-2=2.$$

Example 2. Solve $\frac{x+2}{3\frac{1}{2}}-\frac{\frac{2}{3}(3-x)}{4\frac{1}{4}}=0.$

Now $\frac{x+2}{3\frac{1}{2}}\equiv\frac{2(x+2)}{7}$, and $\frac{\frac{2}{3}(3-x)}{4\frac{1}{4}}\equiv\frac{8(3-x)}{51}$.

$\therefore$ the equation may be written

$$\frac{2(x+2)}{7}-\frac{8(3-x)}{51}=0.$$

Multiplying both sides by 7×51,

$$102(x+2) - 56(3-x) = 0.$$

$$\therefore\ 102x + 204 - 168 + 56x = 0.$$

$$\therefore\ 158x = -36.$$

$$\therefore\ x = -\tfrac{36}{158} = -\tfrac{18}{79}.$$

NOTE ON CHECKS

10·2. It cannot be too strongly emphasised that to check the solution of an equation it is essential to find the value of each side of the equation *as given*. If an intermediate step is checked, it merely shows that no mistake has been made in the subsequent work.

Further, the work in the check should not merely follow the lines of the work of the solution, or a mistake is liable to be repeated. Note, in Example 1 above, the work of the check is entirely different from that of the solution: the brackets have been simplified instead of distributed.

In a case like Example 2 above the arithmetic involved in a check is so laborious that it is better to check merely by going over the work again or reworking it.

EXERCISE 10 a (i)

Solve the following equations:

1. $x + \dfrac{x}{2} + \dfrac{x}{3} = 11.$

2. $36 - \dfrac{8x}{9} = 8.$

3. $\dfrac{x}{2} + \dfrac{x}{3} + \dfrac{x}{4} + \dfrac{x}{5} = x - 17.$

4. $\dfrac{3x}{4} + 5 = \dfrac{5x}{6} + 2.$

5. $\dfrac{x}{2} + \dfrac{x+1}{7} = x - 2.$

6. $\dfrac{x+3}{3} - \dfrac{2x-3}{2} = x - \dfrac{5}{6}.$

7. $\dfrac{3x-1}{5} - \dfrac{2x-3}{3} = 1.$

8. $\dfrac{4x-1}{3} - \dfrac{3x-4}{4} = 6 - \dfrac{x+2}{2}.$

9. $\dfrac{x+7}{4} - \dfrac{2(x+1)}{10} - \dfrac{3x-22}{5} = 1.$

10. $\dfrac{10x+3}{3} - \dfrac{3x-1}{5} = x - 2.$

11. $\dfrac{5x-7}{2} - \dfrac{2x+7}{3} = 3x - 14.$

12. $\frac{5x-2}{3}-\frac{x-8}{4}-\frac{x+14}{2}+2=0.$

13. $\frac{x+4}{3}-\frac{x-4}{5}=2+\frac{3x-1}{15}$

14. $\frac{1}{2}(27-x)=9+\frac{1}{10}(7x-54).$

15. $\frac{x-1}{7}+\frac{23-x}{5}=2-\frac{4+x}{4}$

16. $\frac{5x}{3}+2x+6\left(x-\frac{x}{3}-\frac{4x}{9}\right)=450{,}000.$

17. $\cdot3x+\cdot7x=1.$

18. $\cdot5(x-10)-\cdot4(5-x)=0.$

19. $\frac{x-1\cdot4}{2}-\frac{\cdot7-x}{3}=0.$

20. $10(\cdot2x-\cdot7)-\frac{x}{2}=0.$

21. $\frac{2x-7}{5\frac{1}{4}}+\frac{\frac{1}{2}(3-x)}{\frac{1}{3}}=0.$

22. $\frac{x-3\frac{1}{3}}{2}-2\frac{1}{4}(2x-7)=0.$

Exercise 10 a (ii)

Solve the following equations:

1. $1-\frac{2x}{3}+\frac{3x}{4}=0.$

2. $\frac{x}{3}-\frac{1}{3}-\frac{x}{4}+\frac{1}{4}=\frac{x}{5}-\frac{1}{5}+\frac{1}{6}-\frac{x}{6}.$

3. $\frac{x}{2}+\frac{3x}{4}-\frac{5x}{6}=15.$

4. $\frac{7x}{8}-5=\frac{9x}{10}-8.$

5. $2x-\frac{19-2x}{2}=\frac{2x-11}{2}$

6. $\frac{x+2}{4}-\frac{x+3}{3}+\frac{2x-3}{6}=0.$

7. $5-\frac{3x+1}{4}=\frac{x-10}{6}$

8. $\frac{2x-1}{5}-\frac{x+3}{2}=\frac{3x-5}{5}.$

9. $\frac{3(x-1)}{5}-\frac{2x-5}{2}=1-\frac{3(x-3)}{6}$

10. $x+\frac{3x-9}{\ }=11-\frac{5x-12}{\ }$

11. $\dfrac{2(x-4)}{5} - \dfrac{2x-3}{10} = \dfrac{4x+1}{2} + 1.$

12. $\dfrac{2x-5}{3} - \dfrac{5x-3}{4} + 3 + \dfrac{2x}{3} = 0.$

13. $\frac{1}{7}(3x-4) + \frac{1}{3}(5x+3) = 43 - 5x.$

14. $\frac{1}{6}(8-x) + x - 1\frac{2}{3} = \frac{1}{2}(x+6) - \dfrac{x}{3}.$

15. $\dfrac{5}{6}\left(x - \dfrac{1}{2}\right) - \dfrac{2}{3}\left(\dfrac{x}{2} - 1\right) = 4\frac{5}{6}.$

16. $1{\cdot}25x = 6{\cdot}25.$

17. $1{\cdot}4x - 1{\cdot}1x = {\cdot}3.$

18. $1{\cdot}7(x-2) - {\cdot}3(2x+1) = {\cdot}8.$

19. $\frac{1}{3}({\cdot}5x - {\cdot}4) - \frac{1}{4}(x - 1{\cdot}5) = 2.$

20. $\dfrac{4\frac{1}{2}(2-x)}{3} - \dfrac{2(2x-\frac{1}{3})}{4} = 0.$

21. $\dfrac{x}{\frac{3}{4}} - \dfrac{x-1}{\frac{2}{3}} + \dfrac{2\frac{1}{3}x}{2} = 0.$

22. $\dfrac{x - 5\frac{1}{4}}{3} - 2\frac{1}{3}(2x-3) = 0.$

EXERCISE 10 b (i)

1. Heap. Its one-seventh and its whole make 19. What is the heap? (Ahmes, between 1700 B.C. and 1100 B.C.)

2. Demochares has lived a fourth of his life as a boy; a fifth as a youth; a third as a man; and has spent 13 years in his dotage. How old is he? (From the Collection of Problems by Metrodorus, circ. 310 A.D.)

3. Find the age at which the mathematician Diophantus of Alexandria died from his epitaph, which was to the following effect: "Diophantus passed $\frac{1}{6}$ of his life in childhood; $\frac{1}{12}$ in youth, and $\frac{1}{7}$ more as a bachelor; 5 years after his marriage was born a son, who died 4 years before his father, at half the age his father finally reached".

---

4. A train takes 20 minutes longer to do a journey when it is running at 27 miles per hour than when it is running at 30 miles per hour. What is the length of the journey?

5. I walk at $3\frac{1}{2}$ miles an hour to a town, wait twenty minutes while my bicycle is being mended, and ride back at $10\frac{1}{2}$ miles an hour, the whole journey occupying 2 hours. How far off is the town?

6. A man who bicycles at 9 miles an hour takes 14 minutes longer to go from a village V to a town T than a bus which averages $12\frac{1}{2}$ miles an hour. Find the distance from V to T.

7. A and B start on a certain journey together, A walking 5 miles an hour, B riding 8 miles an hour. B stops for half an hour on the journey but finally arrives at his destination 1 hour before A. What is the length of the journey?

8. A man walks up a mountain at the rate of 2 miles an hour and down again by a way 6 miles longer at the rate of $3\frac{1}{2}$ miles an hour. He is out eight hours altogether. How far has he walked?

---

9. In an examination paper one boy A got three marks more than half of the full marks, and another boy B got six marks less than one-third of the full marks. The marks obtained by A were twice as many as those obtained by B. What were the marks they each obtained?

10. I have a certain number of pennies to divide equally amongst 15 boys; if the number of pennies and the number of boys were each increased by one, each boy would receive 7 pennies less. How many pennies have I to distribute?

11. When A and B sit down to play, B has two-thirds as much money as A. After a while A wins 15*s.*, and then he has twice as much money as B. How much had each at first?

12. It used to cost 1*d.* a mile to travel third class and $1\frac{1}{4}$*d.* a mile to travel second class. I found that by travelling third class instead of second, I saved enough to pay for my lunch, price 2*s.* 6*d.* How long was the journey?

13. A and B go for a holiday together, A starting with £15 more than B. At the end of a week A has spent $\frac{1}{4}$ of his money and B $\frac{1}{3}$ of his, and together they have £58 left. Find what each started with.

14. A man buys a case of oranges at 8*d.* a dozen. He finds 54 spoiled, and, selling the rest at 7 for 5*d.*, he loses 2*s.* 6*d.* on the whole. How many did he buy?

15. At a certain concert 80 tickets were sold, some at 3*s.* 6*d.* each, the rest at 2*s.* 6*d.* each. The sum brought in by the sale of the tickets was £11. 10*s.* 0*d.* How many of each kind were sold?

16. Three-quarters of the coins in a bag are sovereigns, 12 are half-sovereigns and the rest half-crowns. If the total value of the coins is £67, find the total number of coins in the bag.

17. A man leaves home with a certain sum of money in his pocket; he spends one-eighth of it in travelling expenses, one-half of the remainder in purchases, and the rest, amounting to 21*s.*, he loses. How much did he start with?

18. What number must be added to the numerator and also to the denominator of $\frac{3}{7}$ so that the result may equal $\frac{2}{3}$?

19. How soon after noon will the hands of a clock be together again? (In x minutes, how many minute spaces will each hand have traversed? What is the difference in the number of spaces traversed by the time the hands coincide again?)

Exercise 10 b (ii)

1. Heap. Its two-thirds, its one-half, its one-seventh and its whole make 33. What is the heap? (Ahmes, between 1700 B.C. and 1100 B.C.)

2. "Tell me, illustrious Pythagoras, how many pupils frequent thy school." "One half", replied the philosopher, "study mathematics, one fourth natural philosophy, one seventh observe silence, and there are three females besides." How many frequented the school?

3. A person being asked what o'clock it was, replied: "The hours of the day which remain are equal to $\frac{3}{4}$ of those elapsed". How many hours have elapsed? [The ancients divided the day into 12 hours.]

---

4. A motor-cyclist on the Bath road rides at 30 miles per hour where the surface is good and at 10 miles per hour where it is bad. Over a certain section of 50 miles his average speed is 20 miles per hour. What are the lengths of good and bad surface?

5. The train starts in 10 minutes and the station is 1 mile away. I walk 4 miles an hour and I run 8 miles an hour. How far must I run?

6. A man bicycles from his house to a post office, x miles away, at 12 miles an hour, and returns home again at 10 miles an hour. The whole journey takes 33 minutes. Find the value of x.

7. A man walked the first half of the distance from A to B at a speed of 4 miles per hour and the second half at a speed of 3 miles per hour. The second half took 18 minutes more than the first. Find the distance from A to B.

8. A man drives to a certain place at the rate of 8 miles an hour. Returning by a road 3 miles longer at the rate of 9 miles an hour, he takes $7\frac{1}{2}$ minutes longer than in going. How long is each road?

9. A lazy boy was told to divide one-half of a certain number by 6, and the other by 8, and then to add the quotients. To save trouble he divided the number itself by 7. Given that his answer is too small by 7, determine the number.

10. A man is engaged for 70 days. He is to receive 2*s.* per day when he works, but is to be fined 6*d.* for every day that he is absent. He receives altogether £5. 2*s.* 6*d.* How many days did he work?

11. The number of seats in a certain class-room (A) is three-quarters of the number in another room (B). If four seats were moved from A into B, B would have twice as many seats as A. Find the number of seats in each room.

12. A third class railway ticket now costs $1\frac{3}{4}d.$ a mile, and before the War it used to cost 1*d.* a mile. For a certain sum of money I used to be able to travel 12 miles further than I can travel now. What is this sum?

13. I rode one-third of a journey at the rate of 10 miles an hour, one-third more at the rate of 9 miles an hour, and the rest at the rate of 8 miles an hour. If I had ridden half the journey at the rate of 10 miles an hour, and the other half at the rate of 8 miles an hour, I should have been half a minute longer on the way. What distance did I ride?

14. A man bought a certain number of eggs at 4 for fivepence. Seven of the eggs were broken, but, the rest being sold at 3 for fourpence, the man only lost twopence altogether. How many eggs did he buy?

15. The prices of the stalls, pit and gallery of a theatre are respectively ten shillings, half-a-crown and one shilling. The pit holds twice as many and the gallery three times as many as the stalls. If the receipts are £90 when all the seats are full, find the number of people present.

16. How many cwt. of copper and zinc, worth £84 and £20 per ton respectively, must be mixed together in order to obtain 14 cwt. of an alloy worth £40 per ton?

17. A man goes to an exhibition, paying 1*s.* for admission. He then spends one-third of what he has left, and afterwards pays 2*s.* 6*d.* for a cab. On reaching home he finds he has half of what he started with. How much had he at first?

18. If a certain number is added to both the numerator and the denominator of the fraction $\frac{3}{13}$, the result is equal to $\frac{4}{9}$. Find the number.

19. At what time between noon and 1 p.m. are the hands of a clock exactly opposite to one another? (See hint to Ex. 10*b* (i), No. 19.)

REVISION PAPERS

Paper 1. (*On Chaps.* 1–6)

1. If $a=7$, $b=5$, $c=2$, $d=1$, find the values of:

(i) $(a-b)\div(c-d)$, (ii) $a-b\div(c-d)$, (iii) $a-(b\div c-d)$,

(iv) $(a-b)\div c-d$, (v) $(a-b\div c)-d$, (vi) $a-b\div c-d$.

2. Simplify, if possible:

(i) $2a\times 1$,	(ii) $2a\times 0$,	(iii) $2a+1$,
(iv) $2a+0$,	(v) $2a\div 1$,	(vi) $0\div 3x$,
(vii) $0\times 3x$,	(viii) $3x\times 0$,	(ix) $0+3x$.

3. Solve the equation

$$2x+3=16-(2x-3).$$

4. Simplify:

(i) $2(a+1)+3(1+a)-4(a-1)$,

(ii) $p(q-r+1)-q(p+r-1)$.

5. Find the angles in fig. P 1.

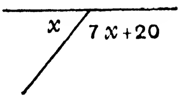


Fig. P 1

6. (i) England batted first and made a runs: Australia then made b runs. In the second innings England made c runs. How many runs must Australia make to win the match?

(ii) How many pence are there in y half-crowns and z shillings, added together?

7. A cyclist travelled for $1\frac{3}{4}$ hours at his normal speed, then for $\frac{3}{4}$ hour at 4 miles per hour above his normal speed. Find his normal speed if the total distance travelled was $30\frac{1}{2}$ miles.

8. Correct the following statements where necessary:

(i) $5a=2$, $\therefore a=\frac{2}{5}$ in., (ii) $\frac{1}{2}ab\equiv\frac{1}{2}a\times\frac{1}{2}b$,

(iii) $\frac{1}{2}(2b)^2\equiv b^2$, (iv) $\frac{2}{3}(2n-4)\times 6\equiv 8(n-2)$.

Paper 2. (*On Chaps.* 1–6)

1. If $a = 1$, $b = 2$, what are the values of:

(i) ab, (ii) $a + b$, (iii) a^b,

(iv) b^a, (v) $a^2 + b^2$. (vi) $2a + 2b$,

(vii) a^a, (viii) b^b, (ix) $a^2b + ab^2$.

2. (i) If $x = 2y$, express $x^2 + y^2$ in terms of x alone.

(ii) If $a = 3b$, find the values of $\dfrac{a^2 + b^2}{a^2 - b^2}$ and $a^2b \div ab^2$.

3. Solve the equations:

(i) $3a + 2 = 2a + 3$, (ii) $3(a + 2) = 2(a + 3)$.

4. Simplify $8a - (5a - 12b + c) + (8a + 7b - c)$ and find its value if $a = 1$, $b = 0$, $c = 1$.

5. Two angles of a triangle are $40 + a$ and $50 - a$ degrees. What can you say about the triangle?

6. (i) I walk $2x$ miles before lunch, and three times as far after lunch; how far do I walk in the day?

(ii) My watch gains 5 seconds every hour; how long will it take to gain t minutes?

7. A has £15 more than B; he gives B £2, and then has twice as much as B. How much had they each at first?

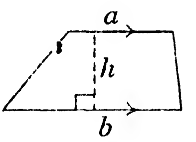


Fig. P 2

8. The area of the trapezium in fig. P 2 can be found from the expression $\frac{1}{2}h(a + b)$. When $a = 10$ and $h = 5$, the area is $57\frac{1}{2}$. What is b?

Paper 3. (*On Chaps.* 1–6)

1. When $r = 1\frac{1}{2}$, $s = 2\frac{1}{2}$ and $t = 0$, evaluate $2sr + 3rs + 7st$.

2. In the expression $\frac{1}{3}\pi r^2 h$, write $r = \dfrac{a}{2}$, $h = \dfrac{3a}{4}$ and simplify.

3. Solve the equation

$$3(169 - x) - (78 + x) = 29x.$$

[*See next page*

4. Simplify, if possible:

(i) $1+a+1+b+ab$, (ii) $a(b-1)+b(a-1)$,

(iii) $c(c-c^2)+c^2(c-1)$.

5. Prove that the fig. P 3 must be a rectangle.

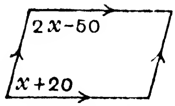


Fig. P 3

6. (i) A 'bus fare is 4 pence; how many journeys can I make for n shillings?

(ii) A car is travelling v miles an hour. How many hours does it take to go half a mile?

7. A man is x years old now; in 12 years' time he will be three times as old as he was 12 years ago: find x.

8. Some of the following statements are identities and the others are equations. Say which are identities. Solve the equations:

(i) $8(r-1)-3(2r-2)=2r-2$,

(ii) $3(x-a)+3(x+a)=0$,

(iii) $y(y^2-1)+(y^2-1)=y^2(y+1)-(y+1)$,

(iv) $9-t^2=t(t+3)-3(3+t)$.

Paper 4. (*On Chaps.* 1–6)

1. Evaluate x^y-y^x when $x=3$, $y=2$.

2. Simplify:

(i) $3(p+q)-2(q-p)$, (ii) $2a^2\times 3a$,

(iii) $y-\dfrac{y+1}{3}$, (iv) $(x^3-x^2)\div x^2$.

3. Solve the following equations and check your solutions:

(i) $1\frac{1}{2}p=9$, (ii) $\frac{1}{2}(q-4)=10$,

(iii) $0=4(\mathrm{R}-1)-3(2-\mathrm{R})$, (iv) $\pi r^2=16\pi\times 4$.

4. From the sum of a^3+6a^2 and $a(5a^2+2a)$ take $5a^3-a^2$.

5. In fig. P 4, two lines are parallel; find x, by drawing a construction line.

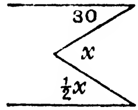


Fig. P 4

6. (i) A sheet of 1*d*. stamps has n rows containing 6 stamps each. Find its cost in shillings.

(ii) Convert a speed of m miles per hour to feet per second.

7. A boat sails a distance north and then half as far again south. After going about and sailing north again for 6 miles, she was 8 miles south of her starting point. How far did she sail north the first time?

8. If $\mathsf{P}+a=b\mathsf{Q}^2$, fill in the spaces:

	P	a	b	Q
(i)	10		$\frac{1}{4}$	8
(ii)	0·6	0·4	9	

Paper 5. (*On Chaps.* 1–6)

1. Find the value of $(a^2+bx)(b^2-cx)$ when $a=3, b=2, c=1, x=0$.

2. Write more compactly, where possible:

(i) $\dfrac{a\times b}{c}$, (ii) $a\times a\times b$, (iii) $a\times b+b\times a$,

(iv) $a+b-ba$, (v) $\dfrac{1}{a}\times\dfrac{2}{a}\times\dfrac{3}{a}$, (vi) $\dfrac{a}{a}+\dfrac{a}{b}+\dfrac{b}{b}$.

3. Solve the equation

$$5(x-11)-3(x+4)=1.$$

4. Simplify:

(i) $(4a+3b)-(5a+b)$, (ii) $(6a+6b)\div 6$, (iii) $6a+6b\div 6$.

5. The angles of a triangle are y, $y+10$, $2y+10$ degrees. Is it right-angled?

6. (i) How many apples are there in p lb. of apples, if the average weight of an apple is w ounces?

(ii) Find the profit when p plants costing q pence per dozen are sold at r pence each.

[*See next page*

7. A has twice as much money as B; but if he were to give B £2, B would have three times as much money as A. How much money has A?

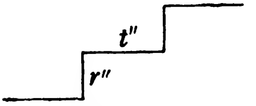


Fig. P 5

8. In many staircases the rise r and the tread t measured in inches are connected by $r=\frac{1}{2}(24-t)$. If $r=5\frac{1}{2}$, what is t?

Paper 6. (*On Chaps.* 1–6)

1. When $a=2$, $b=3$, $c=4$, find the value of

$$\frac{a^2}{(b+c-a)}+\frac{b^2}{(c+a-b)}+\frac{c^2}{(a+b-c)}.$$

2. Simplify each of the expressions $\frac{4}{x}\times\left(\frac{x}{2}\right)^2$, $\frac{4}{x}\div\left(\frac{x}{2}\right)^2$ and find their product.

3. $V=h(b-2h)(l-2h)$. If $h=1$ and $b=4$, then $V=10$. Find l.

4. Simplify $\quad ax(b-y)+bx(a-y)-xy(a+b)$.

5. A triangle is right-angled. The smallest angle is x°. What is the third angle? If this angle is also twice the smallest angle, find x.

6. (i) Find the cost in shillings of $4n$ stamps at $1\frac{1}{2}d.$ each.

(ii) If eggs cost $2d.$ each, how many can be bought for p shillings and sixpence?

7. A is now 20 years older than B. In five years' time, A's age will be double B's age at that time. Find how old A is now.

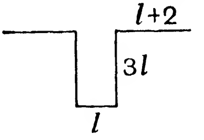


Fig. P 6

8. Three feet of glass tubing is bent into the symmetrical shape indicated in fig. P 6. Find l, lengths shown being in inches.

Paper 7. (*On Chaps.* 1–6)

1. When $n=4$ and $x=3$, what are the values of nx, n^x, x^{n-1}?

2. Simplify, where possible, $x+x^2$, $x\times x^2$, $x\div x^2$, $x-x^2$.

3. Solve the equation

$$4(3+7x)-3(5x-7)=5x+63.$$

4. Find the value of $237a - 237b$, if $a = 252$ and $b = 52$.

5. If A, B, C are the angles of any acute-angled triangle, prove that the sum of $90 - A$, $90 - B$, $90 - C$ is 90.

Can a triangle have angles $\frac{B+C}{2}$, $\frac{C+A}{2}$, $\frac{A+B}{2}$?

6. (i) If a man walks 4 miles an hour, how many miles will he go in x minutes, and how many minutes will it take him to go y yards?

(ii) A candle a in. long burns at the rate of b in. an hour; how many hours has it burned when its length is d in.?

7. A earns twice as many marks as B during the term; but in examination he only gets 200 to B's 300, so that on the total he has $1\frac{1}{2}$ times as many marks as B. How many marks did each get during the term?

8. An aeroplane flies over the course shown in fig. P 7 in 4 hours at an average speed of 200 m.p.h. Find m, which represents the distance AB in miles, and find the other sections of the course.

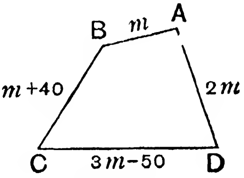


Fig. P 7

Paper 8. (*On Chaps.* 1–6)

1. Show that the equation $x^3 + 11x = 6(x^2 + 1)$ has $x = 3$, 2 and 1 as solutions.

2. Simplify:

(i) $\frac{1}{ab} + \frac{2}{ab} - \frac{3}{ba}$, (ii) $pqr \div rqp$, (iii) $p^2q^2 \div pq$.

3. (i) Solve the equation

$$x - 5 = 1 - \frac{7x}{2}.$$

(ii) Check the solution $x = 0{\cdot}2$ for the equation $6x - 1 = 5x^2$.

4. Simplify the expressions:

(i) $a(b-c) + b(c-a) + c(a-b)$,

(ii) $x(x+7y) - 2y(x-3y) + 5x(x-y)$.

[*See next page*

5. If A, B, C are the three angles of a triangle:

(i) write down the connection between A, B and C,

(ii) find B if it is 10° more than C, and 10° less than A.

6. (i) Pencils cost s shillings a gross and are sold at p pence each. What is the profit, in pence, on every gross?

(ii) If a hens lay b eggs in c days, how many hens must be kept to supply x eggs per week?

7. A father is 35 years old and his son 7 years old; in how many years' time will the father be three times as old as his son?

8. For the staircase in fig. P 8, $r = \frac{1}{2}(24 - t)$.

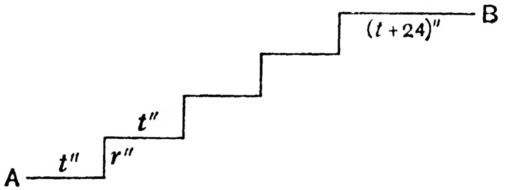


Fig. P 8

(i) If $t = 13$, what is r? What is the height of B above A?

(ii) If 3 yd. of carpet are needed to stretch from A to B, find t.

Paper 9. (*On Chaps.* 1–6)

1. Find the values of $x+6$, $6x$, 6^x, $6-x$, when $x = 2$.

2. Simplify:

(i) $(t^2 \times t^3) \div (t^3 \times t^2)$, (ii) $N \times \frac{1}{2}N \div \frac{1}{3}N$,

(iii) $4(k+\frac{1}{4}) - \frac{1}{4}(k+4)$, (iv) $\frac{1}{2}[x + \frac{1}{2}(x+\frac{1}{2})]$.

3. Solve the equation

$$0{\cdot}5x + 4{\cdot}2(x-1) = 5{\cdot}2.$$

4. Fill in the brackets in

$$3a - 10b + 15c \equiv 3a - 5(\qquad).$$

Simplify

$$(1 - n + 3n^2) - 3n(2 + 4n - 3n^2) + n^2(n - 8).$$

5. In fig. P 9, OP bisects the obtuse angle.

(i) Using brackets, write down an expression for $\angle$ POQ.

(ii) When $\angle$ POQ $=68°$, find a.

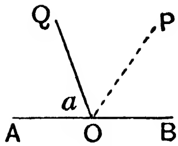


Fig. P 9

6. (i) How many minutes is it before noon at m minutes past 10 o'clock in the morning?

(ii) If x yards of lace cost a shillings and b pence, what will y yards cost? Give a numerical answer if $x=3$, $a=6$, $b=9$, $y=5$.

7. On my way home I go by train to a junction and then either walk 3 kilometres home or wait 15 minutes, travel 10 minutes more by train and then walk a kilometre. I get home at the same time by the two methods. At what speed do I walk (in kilometres per hour)?

8. (i) One solution of the equation $x+\frac{1}{x}=2{\cdot}9$ is $x=\frac{2}{5}$. Verify this.

(ii) By considering the L.H.S. of this equation, write down another solution.

Paper 10. (*On Chaps.* 1–6)

1. Find the values of x^2+y^2, $(x+y)^2$, x^2-y^2, $(x-y)^2$, $(x+y)(x-y)$ when $x=5$, $y=2$.

2. Write the following expressions in their simplest forms: $3x\times 2x$, $3x+2x$, $x^3\times x^2$, x^3+x^2, $3x\div 2x$, $x^3\div x^2$.

3. Solve the equation

$$2(x-4)+3(12-x)=33-2x.$$

4. Simplify $10-\frac{1}{2}[12-(4+2x)]$. Check by writing $x=2$.

5. Using fig. P 9, write down an expression for $\angle$ AOP.

(i) What value of a makes $\angle$ AOP $=110°$?

(ii) What value of a makes $\angle$ AOP $=90°$?

(iii) What value of a makes $\angle$ AOP twice $\angle$ POB?

6. (i) A coil of wire f ft. long weighs w lb. If y yd. are cut off, what is the weight of the remainder?

(ii) A square field measuring c yd. in the side has an area of x acres. Express x in terms of c.

[*See next page*

7. Farnham and Guildford lie on the road from Winchester to London. Farnham is one-third of the way and Guildford half-way. The distance from Winchester to London being x miles, express the distance between Farnham and Guildford in terms of x. If this is 10 miles, find x.

8. To increase the garden in fig. P 10 the area within the dotted line is added. Find x if the new fencing required is 50 ft. long.

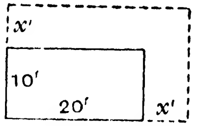


Fig. P 10

Paper 11. (*On Chaps.* 1–6)

1. If $a=5$, $b=4$, $c=3$, which is the greater $\frac{a-c}{b-c}$ or $\frac{a+c}{b+c}$?

2. Simplify:

(i) $abc \div cba$, (ii) R^2+r^2+2Rr when $r=\frac{1}{2}R$,

(iii) $\frac{(y+1)-(y-1)}{(y+1)+(y-1)}$, (iv) $n\left(1+\frac{1}{n}\right)-\frac{1}{n}(n+1)$.

3. Solve the equation

$$3(x-5)+2(x-3)=3(x-4)+3(x-1)-(2x-1).$$

4. Simplify

$$5x+3y+4z-\{x-(y+z)\}-\{3x-2(y+2z)\}.$$

5. Two angles of a convex polygon are right angles, and each of the other angles is 120°. How many sides has the polygon?

6. A man's income is £P a year. £T of this is free of tax, and the remainder is taxed at 5*s.* in the £. Write down expressions for:

(i) the tax, in £, that he pays,

(ii) his net income, after paying tax.

7. A boy finds that he has an hour and a quarter at his disposal between lunch and school, and goes for a cycle ride at 10 miles per hour. Presently his tyre punctures and he is compelled to walk back at 4 miles per hour. He gets in 15 minutes late. How far did he ride before the accident? Could he have been punctual if he had not met with the accident?

8. If the rectangles in fig. P 11 are everywhere of width $\frac{1}{4}$B find the area of the whole figure.

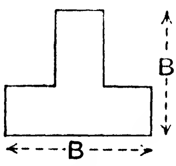


Fig. P 11

Paper 12. (*On Chaps.* 1–6)

1. Write down the number represented by $2x^3+7x^2+3$ when $x=10$; and the number represented by $4+\frac{5}{x}+\frac{9}{x^3}$ when $x=0{\cdot}1$.

2. Verify the identity

$$\left(x+\frac{1}{x}\right)^2-\left(x-\frac{1}{x}\right)^2\equiv 4$$

for $x=1$ and for $x=2$.

3. Solve the equation

$$4\,(2a+0{\cdot}1)-3\,(0{\cdot}5-a)=0{\cdot}7\,(a+8)+0{\cdot}3a.$$

4. Complete the R.H.S. of the identity

$$12\left(l+b+\frac{x}{4}\right)-9\left(b+l+\frac{x}{3}\right)\equiv 3\,(\qquad).$$

5. A pyramid has four more edges than it has corners. How many sides has its base?

6. If a sheets of paper each x in. long and y in. broad weigh b lb., (i) what is the weight of 1 sq. in. of this paper, (ii) what will k sheets of similar paper, each p in. long and q in. broad, weigh?

7. A tourist finds that, if he spends 16*s.* a day, the money at his disposal will enable him to go on for two days longer than if he spent 18*s.* a day. How much money has he?

8. Find expressions for the total edge, total surface and the volume of the solid block in fig. P 12.

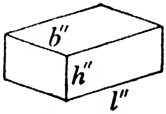


Fig. P 12

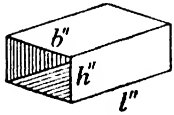


Fig. P 13

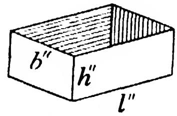


Fig. P 14

Fig. P 13 shows the cover of a match-box. Find an expression for the area of its outer surface.

Find the area of the wood needed to make the open match-box in fig. P 14.

Paper 13. (*On Chaps.* 1–10)

1. Find the value of $(a+x)(b-y)$, when $a=1$, $b=-2$, $x=3$, $y=-3$.

2. Find the sum of

$$x^2+\tfrac{4}{3}xy-\tfrac{1}{3}y^2,\quad \tfrac{2}{3}x^2-xy+\tfrac{1}{3}y^2 \text{ and } \tfrac{4}{3}x^2+\tfrac{1}{3}xy-\tfrac{2}{3}y^2.$$

3. Solve the equations:

(i) $\dfrac{1-a}{a}=\dfrac{2}{3}$, (ii) $\dfrac{1}{b}-1=1$, (iii) $\dfrac{2c-3}{4}=1+\dfrac{c}{3}$.

4. Simplify:

(i) $\dfrac{4a-6b}{2a-3b}$, (ii) $\dfrac{3x-1}{10}-\dfrac{x-3}{15}$, (iii) $12z\left(\dfrac{2-z}{3}-\dfrac{3+z}{4}+\dfrac{2}{z}\right)$.

5. The cost of paper and binding for a certain book used to amount to 10*d.* Since the War the price of paper has been doubled, and the price of binding has been trebled; paper and binding together now cost 1*s.* $11\frac{1}{2}$*d.* for the book. What did the binding cost before the War?

6. A rectangular field measures a yd. by b yd. and has an area of p acres. Give an expression for p in terms of a and b, and use the expression to find p to the nearest whole number when $a=400$ and $b=300$.

7. (i) Verify the identity $\left(x+\dfrac{1}{x}\right)^2-\left(x-\dfrac{1}{x}\right)^2\equiv 4$ for $x=3$, and for $x=-2$.

(ii) Test whether the equation $x^4-10x^2+9=0$ is satisfied by any of the numbers 3, 2, 1, 0, -1, -2, -3.

8. The following table gives the distance a body has fallen from rest after a given number of seconds:

Time in sec.	1	2	3	4	5	6	7
Distance in ft.	16	64	144	256	400	576	784

Plot these points on squared paper and, from your graph, read off

(i) distance a body has fallen after $5\frac{1}{2}$ sec.,

(ii) time a body takes to fall 200 ft.

Paper 14. (*On Chaps.* 1–10)

1. Find the value of $(a^2+bx)(cx-b^2)$ when $a=1$, $b=-1$, $c=2$, $x=-2$.

2. Simplify the expressions:

(i) $x(x+7y)+2y(x-3y)-5x(x-y)$,

(ii) $a(b-c)-b(c-a)+c(a-b)$.

3. Solve the following equation and check your result:

$$\frac{x}{12}-\frac{x+5}{13}-\frac{x-5}{11}+5=0.$$

4. Simplify:

(i) $\dfrac{3x-2y+z}{6x-4y+2z}$, (ii) $\dfrac{2a-3b}{8}-\dfrac{3a+2b}{12}$, (iii) $5uv\left(\dfrac{1}{5}+\dfrac{1}{5u}+\dfrac{5}{v}\right)$

5. The prices of carbon and tungsten electric lamps of the same illuminating power are 11*d.* and 3*s.* 3*d.* respectively. The cost per 100 hours of burning the two lamps is 2*s.* and 8*d.* respectively. For how many hours must the lamps be burnt before the greater economy in burning the tungsten lamp pays for its greater first cost?

6. (i) If 1 lb. of cheese cost x pence, how many shillings will a hundredweight cost?

(ii) If a cart wheel turns a times in b yd., how many times will it turn in c miles?

7. (i) Verify the identity

$$bc(b-c)+ca(c-a)+ab(a-b)\equiv-(b-c)(c-a)(a-b)$$

for $a=-1$, $b=0$, $c=1$.

(ii) Test whether the equation $x^3-4x^2+x+6=0$ is satisfied by any of the numbers 3, 2, 1, 0, −1.

[*See next page*

8. The following table of values for the volume (V) of a constant weight of a gas in a closed space and its absolute pressure (P) was obtained in an experiment on Boyle's Law:

V in c.c.	3	4	5	6	7	8	9	10
P in in. of mercury	108·2	81·1	64·9	54	46·4	40·6	36	32·4

Plot a graph taking V across the page.
From your graph read off

(i) the volume which the gas would occupy when its absolute pressure is that of 60 in. of mercury,

(ii) the absolute pressure of the gas when its volume is 3·6 c.c.

Paper 15. (*On Chaps.* 1–10)

1. If $a=2$, $b=0$, $c=-2$, find the value of $\frac{1}{2}c^3+\frac{1}{3}ab^2-\frac{1}{5}a^2c$.

2. Subtract $7-4x^2$ from 8.
What must be added to $3p^2-2p+5$ to make $4p^2-4p+1$?

3. Solve the following equation and check your result:

$$\frac{2x+1}{7}-\frac{5x-6}{4}=5-\frac{7x+8}{6}.$$

4. Simplify:

(i) $\dfrac{(2x-4y)^2}{x-2y}$, (ii) $\dfrac{2x-y}{3}-\dfrac{3x+2y}{9}-\dfrac{2x-7y}{6}$.

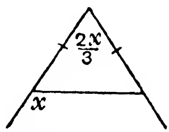


Fig. P 15

5. Find x in the isosceles triangle in fig. P 15.

6. (i) A is x years old and B is y years old. In how many years will the sum of their ages be 100?

(ii) Prove that the sum of two consecutive odd numbers is necessarily a multiple of 4.

7. (i) If $2y=ax^2+1$ and if $y=2$ when $x=1$, find y when $x=3$.

(ii) If $y=3x$, express $\dfrac{y^2}{4}+\dfrac{x^2}{4}+\dfrac{xy}{2}$ in terms of x.

8. The following observations were made in some experiments in towing a canal boat:

v, the speed in m.p.h.	1·7	2·4	3·2	3·6	4
P, the pull in lb.	80	150	250	310	370

Plot the graph and from it estimate (i) P, when speed is 2·6 m.p.h., (ii) v, when pull is 275 lb.

Paper 16. (*On Chaps.* 1–10)

1. If $x=3$, $y=-4$, $z=3$, find the value of

$$(x-y)^2+(y-z)^2-2(x-y)(y-z).$$

2. Simplify each of the following:

(i) $-7-4+2+9$, (ii) $9y-7y-y-8$,

(iii) $P(-3+P)+3(P-1)$, (iv) $-q(-q-1)+q(1-q)$.

3. Solve the equation

$$3x+1-\frac{5x-7}{17}=\frac{x-1}{2}+\frac{7x}{3}.$$

4. Simplify:

(i) $\dfrac{(3x-2y)^2}{6x-4y}$, (ii) $12\left\{\dfrac{2x}{3}-\dfrac{3x}{4}+\dfrac{5x}{6}\right\}$, (iii) $2a-[3\overline{a+x}-4\overline{a-x}]$.

5. A and B have together 17*s.*, A and C have 15*s.*, B and C have 12*s.* What has A?

6. The triangle in fig. P 16 is isosceles.

(i) Write down an expression for angle P.

(ii) Simplify the expression and prove it correct for any three values of a.

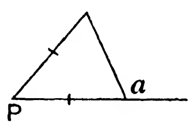


Fig. P 16

7. Given that

$$(a+b)^3=a^3+3a^2b+3ab^2+b^3,$$

write down the expanded forms of $(1+x)^3$, $(1-x)^3$, $(a-b)^3$.

[*See next page*

6. Table giving the height of the barometer at various heights above sea-level:

Height above sea-level in ft.	0	2000	4000	6000	8000	10,000	12,000
Height of barometer in in.	30	27·8	25·7	23·8	22·1	20·5	19

Show the above in a graph and from it read off the height of the barometer at an altitude of 3000 ft. and 6400 ft. Also the altitudes when the readings of the barometer are 20 in. and 24·4 in.

Paper 17. (*On Chaps.* 1–10)

1. Find the value of

$$(b-c)^3+(c-a)^3+(a-b)^3-3(b-c)(c-a)(a-b)$$

when $a=2$, $b=1$, $c=-1$.

2. Find c in terms of a:

(i) when $c^2=a^2+\left(\frac{2a}{3}\right)^2-2\times a\times\frac{2a}{3}$,

(ii) when $c^2=a^2+\left(\frac{2a}{3}\right)^2+2\times a\times\frac{2a}{3}$.

3. Solve the equation

$$\frac{5-x}{2}+\frac{2-x}{5}-\frac{x-1}{10}=0.$$

4. Express as a single fraction

$$\frac{2x-3}{4}-\frac{3x-4}{6}+\frac{x-7}{3}.$$

5. The route of one railway between two towns is 7 miles longer than that of another. A train on the first railway averaging 50 miles an hour takes 3 minutes longer to do the journey between the two towns than a train on the second railway averaging 48 miles an hour. What are the lengths of the two routes?

6. (i) If a train travels x miles in t hours, how many ft. does it cover in s sec.?

(ii) Posts are placed along a railway line at intervals of a yards. An observer in a train notes the time between passing the first and the nth of these posts to be t sec. Express the speed of the train in miles per hour.

7. (i) If $y=a(x-1)(x-2)$ and if $y=4$ when $x=3$, find y when $x=4$.

(ii) If $a=\frac{x}{2}$ and $b=\frac{x}{3}$, express a^2+b^2 in terms of x.

8. The table shows the annual premiums payable for life insurance policies of £1000 taken out at various ages:

Age next birthday	25	30	35	40	45	50	55	60
Annual premium	13·9	15·8	18·5	21·9	26·5	32·7	41·3	53·0

Illustrate these graphically. Find from the graph the premiums at ages 32, 53, 23, 65.

Paper 18. (*On Chaps.* 1–10)

1. Find the value, when $u=-6$, $t=2$, $a=3$, of (i) u^2, (ii) $u+at$, (iii) $ut+\frac{1}{2}at^2$; and, if $v=u+at$, find the value of u^2-v^2, using the same data as before.

2. Simplify:

(i) $P-Q-(P-Q)$, (ii) $(Q-P)-(Q+P)$,

(iii) $P-Q-(P+Q)$, (iv) $(Q-P)-(P-Q)$.

3. Solve $\frac{3{\cdot}2x+2{\cdot}1}{17}=\frac{3{\cdot}4x+5{\cdot}6}{14}$, giving x to the nearest tenth.

4. Simplify $15\left\{\frac{2x-3y}{3}-\frac{3x-5y}{5}+x\right\}$.

5. The area shown in fig. P 17 is $33\frac{3}{4}$ sq. in. Find x.

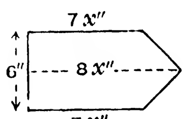


Fig. P 17

[*See next page*

6. A candle, which to begin with is l inches long, after burning for t minutes is x inches long. Find a formula for the total number of minutes that the candle will burn.

7. $1+2+3+\ldots+n\equiv\frac{1}{2}n(n+1)$. Verify this for $n=7$ and for $n=9$.

8. The following results were obtained with the double-sheaved pulley block:

Load in lb.	1	2	3	4	6	8	10	12
Efficiency %	51	65	72	76	81	84	85	85

Show graphically the relation between load and efficiency.

From the graph find the probable efficiency for loads of $2\frac{1}{2}$ lb. and 5 lb.

Paper 19. (*On Chaps.* 1–10)

1. If $y=(1+x)(1-x+x^2)$, find y when (i) $x=-\frac{1}{2}$, (ii) $x=-2$.

2. Simplify

$$x^2(x+3)-2x(x^2-x+3)-(x^3-2x^2-3).$$

Check by putting $x=1$.

3. If the following equation is satisfied by the value $x=2$, find the value of a.

$$\frac{x-a}{3}-\frac{2x-3a}{2}=\frac{a}{6}.$$

4. Simplify $$\frac{4x-3}{3}-\frac{4x-2}{10}-\frac{x-4}{5}.$$

5. In a certain building there are four flats. The height of each flat is 3 ft. more than half the height of the flat below. The top flat is half the height of the bottom flat. Find the height of each flat in ft.

6. In a form consisting of m boys the average mark was a, and in another form of n boys the average mark was b. What was the average mark of the whole number of boys?

What should have been the average mark in the first form in order to make the average mark for the whole equal to c, the average mark in the second form being unaltered?

7. Use the identity $a^3+b^3=(a+b)(a^2-ab+b^2)$ to express a^3+1, a^3+8, a^3-b^3 in forms corresponding to the right-hand side of the identity.

8. The table shows the compound interest on £100 at 5 % per annum:

No. of years	2	4	6	8	10	12
Comp. Int. in £	10·2	21·6	34·0	47·7	62·9	79·6

Draw a graph to illustrate these figures; on the graph show also the simple interest for these times.

From the graph read off the compound interest for 7 and for 11 years.

Also on the same axes draw a graph to show the excess of compound interest over simple interest. When is the excess first as much as £10?

Paper 20. (*On Chaps.* 1–10)

1. If $p=2$, $q=-1$, $r=3$, evaluate:

(i) $p+q+r$, (ii) $2p+3q-4r$, (iii) $p-q$,

(iv) $r-p$, (v) $pq+qr$, (vi) rqp.

2. Take x^3+3x^2+3x+1 from x^3-3x^2+3x-1.
From the sum of a^3+6a^2 and $a^2(5a+2)$ take $5a^3-a^2$.

3. Solve $$3x-4-\frac{4(7x-9)}{15}=\frac{4}{5}\left(6+\frac{x-1}{3}\right).$$

4. Simplify:

(i) $\frac{a}{x}\times\frac{x+a}{a^2}\times\frac{a}{a+x}$, (ii) $1-\frac{1}{a}-\frac{1}{2a}$,

(iii) $a-\frac{1}{a}-\frac{2}{a}$, (iv) $1-\frac{1-a}{a}$.

5. A farmer bought a flock of sheep at 35*s.* each. He has to sell five-eighths of the flock for 30*s.* each and the remainder at 25*s.* each. If he lost £13. 15*s.*, how many sheep did he buy?

[*See next page*

6. An aeroplane travels over a course with the wind at p m.p.h. and against the wind at q m.p.h. Find its average speed for the double journey.

7. In the expression $4+3y-2xy+x^2$ write $\frac{x-1}{2}$ for y.

If the value of the result is 4, find x and then find y.

8. Use the following table to draw a graph to show how the temperature of a healthy person changes during the day:

Time	3 a.m.	6 a.m.	9 a.m.	Noon
Temp. in ° C.	36·45	36·55	36·70	36·95
Time	3 p.m.	6 p.m.	9 p.m.	Midnight
Temp. in ° C.	37·30	37·25	36·95	36·60

Write a short description of what the graph shows.
Taking normal temperature as 37° C., between what hours is temperature above normal?

Paper 21. (*On Chaps.* 1–10)

1. If $y=x(x-1)(x-2)$, find y when $x=-1, -2, +2$.
What other value of x would give the same value for y as $x=+2$?

2. Add $2x-3y+1$, $3x+7y-10$, $-4x+y+3$.
Check by putting $x=y=1$.

3. Solve the equation

$$\tfrac{1}{14}(3x+\tfrac{2}{3})-\tfrac{1}{7}(4x-6\tfrac{2}{3})=\tfrac{1}{2}(5x-6).$$

4. What must be added to $\frac{x+y}{2}$ to give $\frac{x+y}{3}$?

What must be subtracted from $\frac{2W-30}{5}$ to give $\frac{W+30}{10}$?

5. A man has £1000 invested partly at 4%, partly at 5%. His total income is £48. Find the amounts of his investments.

6. A closed box with a lid is made of wood x in. thick. The outside length, breadth and depth are a, b, c in. respectively. Write down expressions for the inside length, breadth and depth. Give expressions for (i) the whole volume (wood and inside volume together), (ii) the inside volume, (iii) the volume of wood.

7. Given that

$$(a+b)^4 \equiv a^4+4a^3b+6a^2b^2+4ab^3+b^4,$$

write down the expanded forms of $(x+1)^4$, $(x-1)^4$, $(a-b)^4$.

8. The data refer to the progress of an electric supply company. Draw graphs showing the number of units sold and the number of customers connected in the various years.

Year	Millions of units sold	No. of customers
1926	73·6	11,366
1928	133·6	20,577
1929	165·6	26,181
1930	174·5	33,253
1931	175·4	41,860
1932	213·7	51,533

(i) From the graph find approximate figures for 1927.

(ii) Make general statements about the changes in the two sets of numbers.

Paper 22. (*On Chaps.* 1–10)

1. Find the value of

$$\frac{3y}{x-y}-\frac{x}{x+y}-\frac{x^2+2xy}{2xy-y^2},$$

when $x=2$, $y=-\frac{1}{2}$.

2. If $\mathsf{X}=90-a$, $\mathsf{Y}=90-b$, $\mathsf{Z}=a-b$, find in terms of a and b:

(i) $\mathsf{X}-\mathsf{Y}-\mathsf{Z}$, (ii) $\mathsf{X}-(\mathsf{Y}-\mathsf{Z})$, (iii) $(\mathsf{X}-\mathsf{Y})-(\mathsf{X}-\mathsf{Z})$.

3. Solve the equation

$$\frac{3x+1}{4}-\frac{2x-3}{7}=\frac{4x-3}{8}-\frac{3x+1}{14}.$$

[*See next page*

4. Simplify

$$\frac{2(x-y)}{3}-\frac{3(x-2y)}{4}.$$

Use the result to *write down* the results of simplifying:

(i) $\frac{2(x-1)}{3}-\frac{3(x-2)}{4}$, (ii) $\frac{2(q-p)}{3}-\frac{3(q-2p)}{4}$

5. A costermonger sells half of his oranges *plus* half an orange; again he sells half of what remains *plus* half an orange; and so again a third time. His store is now exhausted; how many did he start with?

6. Fig. P 18, in which all the angles are right angles, represents a piece of cardboard which can be folded to form a closed rectangular box.

(i) Prove that $d=2b+c$.

(ii) Find an expression for the volume of the box in terms of a, b, c.

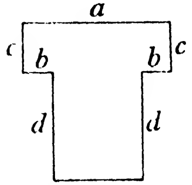


Fig. P 18

7. $$1^2+2^2+3^2+\ldots+n^2\equiv\tfrac{1}{6}n(n+1)(2n+1).$$

Verify this for $n=5$ and for $n=7$.

8. The following data refer to the American cotton crop:

Year	1922	1923	1924	1925	1926	1927
Yield in lb./acre	125	141	131	157	167	183
Price in cents/10 lb.	18·1	25·8	30·1	24·2	19·7	14·4
Year	1928	1929	1930	1931	1932	
Yield in lb./acre	155	153	155	148	201	
Price in cents/10 lb.	19·7	18·7	15·3	9·90	5·88	

Make general statements about the changes in the two sets of numbers and the connection between them.

END OF PART ONE

ANSWERS

Exercise 1 d

PAGE

7. **7.** $\frac{36c}{100}$. **8.** 6 miles, $3x$ miles. **9.** $60y$ miles.

10. $\frac{z}{t}$ m.p.h. **11.** $\frac{d}{60}$ hr.

12. (i) $\frac{k}{h}m$ miles, (ii) $\frac{s}{m}h$ hr. **13.** $\frac{22}{15}v$.

14. (i) $\frac{ph}{s}$ lb., (ii) $q\frac{Z}{z}$ pence, (iii) £$\frac{20b}{a}$ L.

Exercise 1 e

10. **6.**

Fig.	1·11	1·12	1·13	
Perimeter	$2b+10$	$4a+4t$	$6w+6t$	in.
Area	$5b+a$	a^2+2at	$5wt$	sq. in.

7. $8l^2$, $7s^2$, $16l^2$ sq. in.

11. **8.**

Fig.	1·17	1·18	1·19	1·20
Area	$6x+4$	$12l+12t+2lt$	$2lb+2lh+2bh$	$38x^2$ sq. in.
Vol.	$2x$	$6lt$	blh	$12x^3$ cu. in.

Exercise 1 f

12. **7.**

Cost Price	Selling Price	Gain	Gain %
		$s-c$	$100\frac{s-c}{c}$
	$p+g$		$100\frac{g}{p}$
$x-y$			$100\frac{y}{x-y}$
	$a+\frac{ab}{100}$	$\frac{ab}{100}$	
$100\frac{q}{r}$	$q+100\frac{q}{r}$		
$\frac{100s}{100+t}$		$\frac{st}{100+t}$	

PAGE

12. **8.** £$\frac{20}{21}$ P. **9.** $\frac{P}{S}(100+g)-100$. **14.** $\frac{xy}{x+y}$.

EXERCISE 3 b

19. **1.** 17. **2.** 5. **3.** 5. **4.** 1. **5.** 27.
6. 17. **7.** 9. **8.** 6. **9.** $4\frac{1}{2}$. **10.** $1\frac{1}{2}$.
11. 103. **12.** 5. **13.** 8. **14.** 27.
20. **15.** 17. **16.** 31. **17.** 11. **18.** 7. **19.** 12.

EXERCISE 3 d (i)

21. **1.** 3. **2.** $2\frac{1}{2}$. **3.** 4. **4.** 2. **5.** 11.
6. $6\frac{1}{2}$. **7.** 2. **8.** $3\frac{1}{2}$. **9.** 2. **10.** $3\frac{1}{2}$.
11. 1. **12.** 9. **13.** $2\frac{1}{6}$. **14.** $1\frac{2}{3}$. **15.** $\frac{1}{8}$.

EXERCISE 3 d (ii)

1. 5. **2.** 14. **3.** $1\frac{1}{2}$. **4.** $4\frac{1}{2}$. **5.** 4.
6. $4\frac{1}{2}$. **7.** 3. **8.** 2. **9.** 6. **10.** $2\frac{1}{3}$.
11. 10. **12.** 0. **13.** 6. **14.** 5. **15.** 1.

EXERCISE 3 e

22. **7.** $8\frac{1}{3}$. **8.** $3\frac{9}{25}$. **9.** $\frac{3}{8}$.
10. (i) 16, (ii) 20, (iii) 4, (iv) 18, (v) $\frac{3}{2}$, (vi) $\frac{9}{7}$.

EXERCISE 3 g

24. **3.** $12\frac{1}{2}$, 10. **4.** (i) 24, 8, (ii) 27, 3, (iii) 600, 10.
25. **5.** 9. **6.** 4. **7.** 15. **8.** $1\frac{1}{2}$ in. **9.** 4.
10. 125 cu. in. **11.** $2\frac{1}{4}$. **12.** 16. **13.** 12.

EXERCISE 4 a (i)

26. **1.** A, £72; B, £144; C, £216.
2. A, £26; B, £11. **3.** 3 goals, 6 tries.
27. **4.** Daughter £200, son £100. **5.** 5 ft.

EXERCISE 4 a (ii)

1. A, £84; B, £42; C, £14. **2.** £$32\frac{1}{2}$, £$27\frac{1}{2}$.
3. 33 magazines, 66 newspapers. **4.** £150. **5.** 1 in.

Exercise 4 b (i)

PAGE

29. **1.** 2. **2.** In $2\frac{2}{3}$ hr. **3.** At 11.6 a.m.

Exercise 4 b (ii)

1. 2. **2.** In $3\frac{1}{2}$ hr. **3.** At 2 p.m.

Exercise 4 c (i)

31. **1.** 36. **2.** 7. **3.** 60.
4. 36. **5.** 81, 27. **6.** 8°.

Exercise 4 c (ii)

1. 30. **2.** 25. **3.** 35.
4. 50. **5.** 71°. **6.** 40.

Exercise 4 e (i)

33. **1.** 16, 17. **2.** 34, 36. **3.** 23, 25.
4. 13. **5.** 8*s.* 6*d.* **7.** 60.

Exercise 4 e (ii)

1. 28, 29. **2.** 22, 24. **3.** 17, 19.
4. 28. **5.** 13. **7.** 4.

Exercise 4 g (i)

35. **1.** 2. **2.** 2*s.* 6*d.* **3.** $1\frac{1}{2}$ in., $2\frac{1}{2}$ in.
4. (ii) 7. **5.** 4*d.*
6. 24, 20, 16, 12 years. **7.** $18\frac{2}{3}$.
8. 40. **9.** 9 lb.
36. **10.** 46 m.p.h. **11.** 88, 156. **12.** 45.
14. 2*s.* 4*d.* **15.** 114°, 38°, 28°.
16. (ii) 8. **17.** 150.
37. **18.** 2. **19.** 15*s.* **20.** 10.

Exercise 4 g (ii)

1. 100, 167; 197, 50. **2.** 4*s.* 6*d.* **3.** 2.
4. (iii) $1\frac{1}{4}$. **5.** After $1\frac{3}{7}$ hr. **6.** (iii) 16.
38. **7.** 20. **8.** 70°. **9.** A, 15*s.* 9*d.*; B, 4*s.* 3*d.*
10. $\frac{3}{7}$ acre. **11.** 36; it is a trapezium. **12.** 60.
13. 100°, 20°. **14.** 2. **15.** 2697, 1871.
16. 3 miles from the station.
39. **18.** $42\frac{1}{2}$. **19.** 1800 infantry, 600 artillery, 200 cavalry.
20. 20.

Exercise 5 b

PAGE

42. **1.** (i) $5a+8$, (ii) $13+12b$, (iii) $15c+33d$, (iv) $5\mathsf{W}+5\mathsf{P}$, (v) $3p+2pq+3q$, (vi) $2rs+2rt$, (vii) $3+4x+x^2$, (viii) $5w^2+14w$, (ix) $16x+4$, (x) $2a^2+5ab+3b^2$.
2. (i) 1, (ii) 1, (iii) 1, (iv) 1, (v) $13\frac{1}{2}$, (vi) 2.

Exercise 5 c

43. **2.** (i), (iii) Untrue; (ii), (iv) True.

Exercise 5 e

44. **1.** (i) $9-\mathsf{P}$, (ii) $x-6y$, (iii) t^2-4z^2, (iv) K, (v) $6-3\mathsf{M}$, (vi) $4a-4b$.

45. **4.** (i) 5, (ii) 15, (iii) $\frac{1}{2}$, (iv) $\frac{1}{2}$, (v) $\frac{5}{8}$, (vi) 4.

Exercise 5 g

47. **4.** (i) 4, (ii) 8, (iii) 2, (iv) a, (v) 3, (vi) $a-b$, (vii) $x-y$, (viii) 0, (ix) $2x^2$, (x) $2a$.

Exercise 5 h

49. **4.** (i) $8+2p$, (ii) $x+y-2$, (iii) $9x+8$, (iv) ab, (v) $3x$, (vi) $3rs-2r^2-s^2$, (vii) $5p+13q$.

Exercise 5 i

50. **1.** Identity. **2.** 1. **3.** Identity.
4. $a=2$. **5, 6, 7, 8.** Identities. **9.** 1.

Exercise 5 j

1. (i) 51, (ii) 84, (iii) 0, (iv) 36, (v) 5, (vi) 1, (vii) 12.
2. 55.

51. **6.** (i) $2ab+ac+bc$, (ii) $ac-bc$, (iii) $2s^2-2t^2$, (iv) $60-2x$, (v) $11+k$, (vi) $\mathsf{T}-1-t$, (vii) $3q^3+q^2+2q$, (viii) $10y^3-15y^2-10y+15$, (ix) $3-m$, (x) $2ab-2ac$, (xi) $a+5b$, (xii) 0, (xiii) $5ax+2a$, (xiv) $5x-16$, (xv) z^3-3z^2-z-1, (xvi) 2, (xvii) $c^2d-c^2e+d^2e-d^2c+e^2c-e^2d$, (xviii) n^2, (xix) $2+\frac{4}{n}$, (xx) n^2+3n.
7. (i) 90, (ii) x^2+x-1, (iii) $1+x-x^2$, (iv) $12t-\frac{1}{2}-\frac{1}{2}p$.

Exercise 5 k

PAGE

51. **1.** (i) 110, (ii) 76, (iii) $3\frac{1}{2}$, (iv) $\frac{3}{4}$.
2. (i)–(iii) Yes, (iv) No, (v)–(viii) Yes.

52. **7.** $2l+6$, $7b+27$, $9x+15$. **8.** 21, 3, $3\frac{2}{3}$.
9. $10l-35$, $2t^2-35$. **10.** $10\frac{1}{2}$, $\sqrt{52\frac{1}{2}}$.
11. $140-x$, $350-2a$, $190-3c$. **12.** 20, 115, $23\frac{1}{3}$.

Exercise 6 a (i)

53. **1.** 9. **2.** 9. **3.** $3\frac{1}{2}$. **4.** $2\frac{1}{2}$. **5.** 0.
6. 1. **7.** 4. **8.** 3. **9.** 1. **10.** 120.
11. 3. **12.** 1. **13.** 3. **14.** 13. **15.** 13.

54. **16.** $1\frac{2}{9}$. **17.** 4. **18.** 5. **19.** 4.

Exercise 6 a (ii)

1. 13. **2.** 3. **3.** 12. **4.** 13. **5.** 3.
6. 4. **7.** 2. **8.** 4. **9.** 0. **10.** $\frac{1}{8}$.
11. $1\frac{1}{2}$. **12.** 3. **13.** 2. **14.** $\frac{5}{21}$. **15.** 6.
16. $3\frac{1}{3}$. **17.** 3. **18.** $\frac{43}{30}$. **19.** 0.

Exercise 6 b (i)

56. **1.** 7. **2.** 33. **3.** 30. **4.** 10.
5. 17 at 2*s*., 22 at 2*s*. 6*d*.

57. **6.** 90 of 2 cwt., 60 of 5 cwt. **7.** 6 hr. **8.** 6 p.m.
9. $1\frac{1}{18}$ hr. after noon.

Exercise 6 b (ii)

1. 40, 15. **2.** 15. **3.** 12.
4. 46 half-sovereigns, 19 shillings. **5.** 8.
6. $2\frac{1}{2}$ doz. at 3*d*., $7\frac{1}{2}$ doz. at 4*d*. **7.** $1\frac{1}{3}$ hr.

58. **8.** 3.20 p.m., $46\frac{2}{3}$ miles. **9.** $66\frac{2}{3}$ m.p.h.

Exercise 6 c (i)

1. 2. **2.** 5. **3.** £53. 6*s*. 8*d*.
4. A, £87; B, £43. 10*s*. **5.** 11.
6. 3*s*. 6*d*., 2*s*. 6*d*., 1*s*. 6*d*. **7.** 80.

59. **8.** £1100 at 6%, £500 at 7%. **9.** $\frac{1}{2}$ hr.
10. 11. **11.** 6. **12.** 23.

Exercise 6 c (ii)

PAGE

59. **1.** 6*s.* 3*d.*, 3*s.* 9*d.* **2.** £5.
3. Horse £25, cow £17. **4.** 17 florins, 13 half-crowns.
5. 300. **6.** 2*d.* each.
60. **7.** 42 ord., 24 hon. **8.** 3.
9. $1\frac{1}{2}$ hr. **10.** 160, $\frac{3}{4}$ in.; 40, 1 in.
11. 735. **12.** 16; 52°, 52°, 76°, 104°.

Exercise 7 a

62. **2.** Highest July, Lowest Jan.

Exercise 7 b

63. **1.** Near, Sept, Feb.; above, April to Aug. and Nov.
64. **2.** Temp. in Isle of Wight less extreme. Surrounded by sea.

Exercise 7 c

66. **1.** 10·8 ft., 7·2 ft.
67. **2.** 12.48, 6.0. **3.** $128\frac{4}{7}°$, 140°.

Exercise 7 e

69. **1.** (i) 0, 2, 4, ..., 14, (ii) 5, 10, 15, ..., 40,
(iii) 40, 60, 80, ..., 180, (iv) 200, 300, 400, ..., 900,
(v) '61, '71, '81, ..., '31, (vi) 1, 1·5, 2, ..., 4·5,
(vii) 10, 11, 12, ..., 4, (viii) 0, 5, 10, ..., 35,
(ix) 0, 1, 2, ..., 7,
(x) ·005, ·006, ·007, ..., ·012,
(xi) 5000, 7000, 9000, ..., 19,000,
(xii) 180, 200, 220, ..., 320.

Exercise 7 f (i)

70. **1.** 1887, 4·63, 2·8 millions; 1914, 7·7, 4·75 millions.
2. 16 yd., 48 yd.
3. Catches, greatest 1929, least 1921; Value, greatest 1920, 1927; 1923.
71. **4.** British up to 220 yd. and beyond 880 yd.; German up to 305 yd. and beyond 820 yd.
5. In 1920, value up, tonnage down; in 1921, big drop in value, small drop in tonnage.
6. 5800 tons (very roughly); $15\frac{3}{4}$ ft.

EXERCISE 7 f (ii)

PAGE

72. **1.** 63 miles, 1400 ft.

2. (i) 8·7 in., 9·9 in., (ii) 10.24, 1.48, (iii) 12.6.

73. **3.** In 1929, receipts up, expenditure down; 1930, 1931, receipts down, expenditure up.

4. 1906, £260,000, 18·2 millions; 1914, £330,000, 22·4 millions. Revenue has risen steadily, consumption went down 1920–25.

5. Sudden change at about 32°, rises before that, goes down afterwards. 14, 31·9.

74. **6.** 9.48 a.m., 2.0 p.m.; Sept. 4th to 8th.

EXERCISE 7 g

1. (ii) Jan., 5; Feb., 40; Mar., 40; Ap., 20, (iii) Feb. 12th.

2. (i) Sunshine graph, no sun; rain graph, no rain.

75. **3.** 1921, 1927.

77. **5.** The untrained man earns the larger salary till about 24 or 25 years of age, but afterwards earns less.

6. Capital invested has increased faster and faster. Consumption increased steadily till 1912 and then increased much more rapidly.

78. **7.** It would be expected that the H.P. would increase continually with the speed. But it appears that, if a speed of about $17\frac{1}{2}$ knots can be attained, a further increase of speed (up to about $19\frac{1}{2}$ knots) calls for actually less H.P. After $19\frac{1}{2}$ knots the H.P. again increases with the speed. (The existence of the "critical" speed of $17\frac{1}{2}$ knots arises from the shallowness of the water.)

8. (i) Upper, (ii) middle, (iii) lower. Temperature below ground affected less by sunshine.

EXERCISE 8 f

91. **5.** (i) 1, (ii) -1, (iii) ± 1.

92. **8.** (i) -4, (ii) $2\frac{1}{2}$, (iii) -2, (iv) -4, (v) $-1\frac{1}{3}$, (vi) 0, (vii) $-\frac{2}{5}$, (viii) ± 3, (ix) -1, (x) $-\frac{1}{6}$.

EXERCISE 8 g (i)

1. $10\frac{1}{2}$.	**2.** $-\frac{7}{8}$.	**3.** 0.	**4.** 0.
5. $\frac{5}{21}$.	**6.** -2.	**7.** $1\frac{1}{4}$.	**8.** $1\frac{2}{9}$.
9. -4.	**10.** $2\frac{1}{2}$.	**11.** $-4\frac{1}{6}$.	**12.** $-\frac{2}{5}$.
13. 5.	**14.** $1\frac{14}{31}$.	**15.** $-5\frac{4}{5}$.	**16.** 0.

EXERCISE 8 g (ii)

PAGE

93. **1.** $2\frac{2}{3}$. **2.** $-2\frac{1}{2}$. **3.** $1\frac{11}{16}$. **4.** 4.
5. $\frac{4}{5}$. **6.** $1\frac{1}{3}$. **7.** 5. **8.** $-2\frac{1}{2}$.
9. $4\frac{2}{3}$. **10.** $-6\frac{2}{3}$. **11.** 2. **12.** $3\frac{1}{3}$.
13. $1\frac{4}{7}$. **14.** 4. **15.** $2\frac{12}{31}$. **16.** 0.

EXERCISE 8 h

94. **1.** $8x-7y-9$. **2.** $8p-11$. **3.** $6+x-2x^2$.
4. $-2a-b+c$. **5.** $3x+20$. **6.** $7+2x+6x^2$.
7. $7+5y+y^2$. **8.** z^3+2z^2-1.
9. (i) x^2+3x+5, (ii) y^2-5, (iii) $-pq-q^2$.
10. Rows $2x+2$, $-x^2+11x-2$, x^3-x^2-4x+1.
Columns x^3-x^2+x+1, $6x+2$, $-x^2+2x-2$.
Check x^3-2x^2+9x+1.

EXERCISE 8 i

95. **1.** $-x+y$, $5a-3b$, $-x+2y+4$. **2.** $1-x$.
3. $2a-b+3c$, $-3y^2+2y-6$.
4. x^3-3x^2-3x-2. **5.** $2x^2+xy$.
6. (i) $1-p^3$, (ii) $a+5x+6$,
(iii) $3a^2+3ab+b^2$, (iv) $-x^2+xy+y^2$.
7. $-3a^2+3a$.

EXERCISE 9 d (i)

102. **1.** (i) $\frac{7a}{15}$, (ii) $\frac{7b}{6}$, (iii) $\frac{1}{2c}$.
2. (i) $\frac{y-x}{3xy}$, (ii) $\frac{4q^2+4}{q}$, (iii) $\frac{bx+ay}{ab}$.
3. (i) $\frac{x+1}{x^3}$, (ii) $\frac{ab^2+ab+a^2}{b^2}$, (iii) $\frac{x^3-4x-8}{x^3}$.
4. (i) $-\frac{5}{4}$, (ii) $\frac{7y-12}{6}$, (iii) $\frac{z+1}{2}$.
5. (i) $-\frac{1}{18}$, (ii) $\frac{y-z}{2}$, (iii) $\frac{11y}{42}$.
6. $\frac{16a-11b}{30}$. **7.** $\frac{15x-34}{12}$. **8.** $\frac{9a-5c}{24}$.
9. $\frac{-a+2b-c}{6}$. **10.** $\frac{x-19y}{15}$. **11.** $\frac{11b-a}{6}$.

PAGE

102. 12. $\frac{-6x-19}{9}$. 13. $\frac{8x-5}{8}$. 14. $10x-13$.
15. $-a-10b$. 16. $4x-36y$. 17. $10b-20a$.
18. $11p-20q$. 19. $8-8a$.

Exercise 9 d (ii)

103. 1. (i) $\frac{a}{30}$, (ii) $\frac{19b}{20}$, (iii) $\frac{1}{5c}$.
2. (i) $\frac{2b-a}{4ab}$, (ii) $\frac{W^2-4}{2W}$, (iii) $\frac{p^2-q^2}{pq}$.
3. (i) $\frac{b-a}{a^2b}$, (ii) $\frac{1-d-c}{cd}$, (iii) $\frac{9-6y+3y^2}{y^2}$.
4. (i) $\frac{11-19x}{10}$, (ii) $\frac{17y-1}{12}$, (iii) $\frac{6-z}{4}$.
5. (i) $\frac{x-1}{6}$, (ii) 0, (iii) $\frac{-y-41}{36}$.
6. $\frac{17x-39}{60}$. 7. $\frac{8x}{5}$. 8. $\frac{7a-b}{6}$. 9. $\frac{a}{24}$.
10. $\frac{16x-15}{12}$. 11. $\frac{-18x+17y+29}{20}$.
12. $\frac{5y}{28}$. 13. $\frac{4-13x}{12}$. 14. $11x-31$. 15. $3x$.
16. $x+7y$. 17. $-14x$. 18. $30-4x$. 19. $2x+34$.

Exercise 10 a (i)

105. 1. 6. 2. $31\frac{1}{2}$. 3. -60. 4. 36.
5. 6. 6. 2. 7. -3. 8. 4.
9. 9. 10. $-1\frac{11}{13}$. 11. 7.
106. 12. 4. 13. 3. 14. $8\frac{1}{4}$. 15. $-17\frac{25}{27}$.
16. 90,000. 17. 1. 18. 7·77.... 19. 1·12.
20. $4\frac{2}{3}$. 21. $2\frac{39}{47}$. 22. $3\frac{25}{48}$.

Exercise 10 a (ii)

1. -12. 2. 1. 3. 36. 4. 120.
5. 2. 6. 4. 7. 7. 8. -1.
9. 6. 10. $5\frac{1}{7}$.
107. 11. $-1\frac{5}{9}$. 12. -25. 13. 6. 14. 5.
15. $9\frac{1}{6}$. 16. 5. 17. 1. 18. 4·09....
19. $-21·1$. 20. $1\frac{4}{15}$. 21. $-1\frac{1}{2}$. 22. $1\frac{11}{52}$.

Exercise 10 b (i)

PAGE

107. 1. $16\frac{5}{8}$. 2. 60. 3. 84.
4. 90 miles. 5. $4\frac{3}{8}$ miles.

108. 6. $7\frac{1}{2}$ miles. 7. 20 miles. 8. 22 miles.
9. A, 48; B, 24. 10. 1695. 11. A, 135*s*.; B, 90*s*.
12. 120 miles. 13. A, £48; B, £33.
14. 180.

109. 15. 30 at 3*s*. 6*d*.; 50 at 2*s*. 6*d*. 16. 80.
17. 48*s*. 18. 5.
19. At $5\frac{5}{11}$ min. past 1.

Exercise 10 b (ii)

1. $14\frac{28}{97}$. 2. 28. 3. $6\frac{6}{7}$ hr.
4. $37\frac{1}{2}$ miles, good; $12\frac{1}{2}$ miles, bad. 5. $\frac{2}{3}$ mile.

110. 6. 3. 7. $7\frac{1}{3}$ miles.
8. 15 miles, 18 miles. 9. 2352.
10. 55. 11. A, 18; B, 24.
12. 2*s*. 4*d*. 13. 18 miles. 14. 88.

111. 15. 600. 16. $4\frac{3}{8}$ copper, $9\frac{5}{8}$ zinc.
17. 19*s*. 18. 5.
19. $2\frac{8}{11}$ min. after 12.30.

REVISION PAPERS

Paper 1

112. 1. (i) 2, (ii) 2, (iii) $5\frac{1}{2}$, (iv) 0, (v) $3\frac{1}{2}$, (vi) $3\frac{1}{2}$. 3. 4.
4. (i) $a+9$, (ii) $p-pr-qr+q$. 5. 20″, 160″.
6. (i) $a+c-b+1$, (ii) $30y+12z$. 7. 11 m.p.h.

Paper 2

113. 2. (i) $\frac{5}{4}x^2$, (ii) $\frac{5}{4}$, 3. 3. (i) 1, (ii) 0.
4. $11a+19b-2c$; 9. 5. It is right-angled.
6. (i) $8x$ miles, (ii) $12t$ hr. 7. A, £24; B, £9.
8. 13.

Paper 3

1. $18\frac{3}{4}$. 2. $\frac{\pi a^3}{16}$. 3. 13.

114. 6. (i) $3n$, (ii) $\frac{1}{2v}$. 7. 24.
8. (i), (iii) Identities, (ii) $x=0$, (iv) $t=3$.

Paper 4

PAGE

114. 1. 1. 2. (i) $5p+q$, (ii) $6a^3$, (iii) $\frac{2y-1}{3}$, (iv) $x-1$.
3. (i) 6, (ii) 24, (iii) $1\frac{3}{7}$, (iv) 8. 4. a^3+9a^2. 5. 60.
115. 6. (i) $\frac{1}{2}n$ shillings, (ii) $\frac{22m}{15}$ ft. per sec.
7. 28 miles. 8. (i) $a=6$, (ii) $Q=\frac{1}{3}$.

Paper 5

1. 36. 3. 34. 5. Yes.
6. (i) $\frac{16p}{w}$, (ii) $\left(pr-\frac{pq}{12}\right)$ pence.
116. 7. £3. 4s. 8. 13.

Paper 6

1. $19\frac{4}{5}$. 2. $x, \frac{16}{x^3}, \frac{16}{x^2}$.
3. 7. 4. $2abx-2axy-2bxy$.
5. 30. 6. (i) $\frac{1}{2}n$ shillings, (ii) $6p+3$.
7. 35 years. 8. $3\frac{5}{9}$.

Paper 7

1. 12, 64, 27. 3. $3\frac{3}{4}$.
117. 4. 47,400. 5. Yes.
6. (i) $\frac{x}{15}$ miles, $\frac{3y}{352}$ min., (ii) $\frac{a-d}{b}$.
7. A, 1000; B, 500. 8. $115\frac{5}{7}$, $155\frac{5}{7}$, $297\frac{1}{7}$, $231\frac{3}{7}$.

Paper 8

3. (i) $\frac{4}{3}$. 4. (i) 0, (ii) $6x^2+6y^2$.
118. 5. (ii) 60°. 6. (i) $(144p-12s)$ pence, (ii) $\frac{acx}{7b}$
7. 7. 8. (i) $5\frac{1}{2}$, 22 in., (ii) 12.

Paper 9

PAGE

118. **2.** (i) 1, (ii) $\frac{3}{2}$N, (iii) $3\frac{3}{4}k$, (iv) $\frac{3}{4}x+\frac{1}{8}$. **3.** 2.
4. $(2b-3c)$, $2-8n-14n^2+10n^3$.

119. **5.** (i) $\frac{1}{2}(180-a)$, (ii) 44.
6. (i) $120-m$, (ii) $\frac{y(12a+b)}{x}$ pence; 11*s.* 3*d.*
7. 4·8 km. per hr. **8.** (ii) $\frac{5}{2}$.

Paper 10

1. 29, 49, 21, 9, 21. **3.** 5. **4.** $6+x$.
5. $90+\frac{1}{2}a$, (i) 40, (ii) 0, (iii) 60.
6. (i) $\frac{w(f-3y)}{f}$ lb., (ii) $x=\frac{c^2}{4840}$.

120. **7.** $\frac{1}{6}x$ miles; 60. **8.** 5.

Paper 11

1. The first. **2.** (i) 1, (ii) $2\frac{1}{4}$R^2 or $9r^2$, (iii) $\frac{1}{y}$, (iv) $n-\frac{1}{n}$.
3. 7. **4.** $x+6y+9z$. **5.** 5.
6. (i) £$\frac{1}{4}$(P − T), (ii) £($\frac{3}{4}$P + $\frac{1}{4}$T).
7. $4\frac{2}{7}$ miles, yes. **8.** $\frac{7}{16}$B^2.

Paper 12

121. **1.** 2703, 9054. **3.** 0·67.
4. R.H.S. $=3(l+b)$. **5.** 5.
6. (i) $\frac{b}{axy}$ lb., (ii) $\frac{bkpq}{axy}$ lb. **7.** £14. 8*s.*
8. $4(b+h+l)$ in., $2(hl+lb+bh)$ sq. in., blh cu. in.; $2(hl+bl)$ sq. in.; $(2bh+2lh+bl)$ sq. in.

Paper 13

122. **1.** 4. **2.** $3x^2+\frac{2}{3}xy-\frac{2}{3}y^2$.
3. (i) $\frac{3}{5}$, (ii) $\frac{1}{2}$, (iii) $10\frac{1}{2}$.
4. (i) 2, (ii) $\frac{7x+3}{30}$, (iii) $24-z-7z^2$.
5. $3\frac{1}{2}$ pence. **6.** $p=\frac{ab}{4840}$; 25.
7. (ii) ± 3, ± 1. **8.** (i) 484 ft., (ii) 3·54 sec.

Paper 14

PAGE

123. 1. -15. 2. (i) $-4x^2+14xy-6y^2$, (ii) $2ab-2bc$.

3. 60. 4. (i) $\frac{1}{2}$, (ii) $-\frac{13b}{24}$, (iii) $uv+v+25u$.

5. 175 hr. 6. (i) $\frac{28x}{3}$, (ii) $\frac{1760ac}{b}$.

7. (ii) 3, 2, -1.

124. 8. (i) 5·41 c.c., (ii) 90·1 in.

Paper 15

1. $-2\frac{2}{5}$. 2. $1+4x^2$, p^2-2p-4.

3. 10. 4. (i) $4x-8y$, (ii) $\frac{11y}{18}$.

5. 135. 6. (i) $50-\frac{x+y}{2}$.

7. (i) 14, (ii) $4x^2$.

125. 8. (i) 170, (ii) 3·36.

Paper 16

1. 196. 2. (i) 0, (ii) $y-8$, (iii) P^2-3, (iv) $2q$.

3. 15. 4. (i) $\frac{3x-2y}{2}$, (ii) $9x$, (iii) $3a-7x$.

5. 10*s*. 6. $2(a-90)$.

126. 8. 26·6 in., 23·4 in.; 10,600 ft., 5300 ft.

Paper 17

1. 0. 2. (i) $\pm\frac{1}{3}a$, (ii) $\pm\frac{5}{3}a$.

3. $3\frac{3}{4}$. 4. $\frac{4x-29}{12}$. 5. 115, 108 miles.

127. 6. (i) $\frac{22xs}{15t}$ ft., (ii) $\frac{45a(n-1)}{22t}$ m.p.h. 7. (i) 12, (ii) $\frac{13x^2}{36}$.

8. £16·8, £37·5, £13·2. It is impossible to tell the premium at 65; it is probably between £65 and £75.

Paper 18

1. (i) 36, (ii) 0, (iii) -6; 36.

2. (i) 0, (ii) $-2P$, (iii) $-2Q$, (iv) $2Q-2P$.

3. $-5·1....$ 4. $16x$. 5. $\frac{3}{4}$.

128. 6. $\frac{lt}{l-x}$ min. 8. 69, 78·7 per cent.

Paper 19

PAGE

128. 1. (i) $\frac{7}{8}$, (ii) -7. 2. $-2x^3+7x^2-6x+3$.

3. $\frac{4}{3}$. 4. $\dfrac{11x}{15}$ 5. 14, 10, 8, 7 ft.

6. $\dfrac{ma+nb}{m+n}$; $\dfrac{c(m+n)-nb}{m}$

129. 8. £40·7, £71·0; after 9 years.

Paper 20

1. (i) 4, (ii) -11, (iii) 3, (iv) 1, (v) -5, (vi) -6.
2. $-6x^2-2$; a^3+9a^2. 3. $7\frac{1}{13}$.
4. (i) $\dfrac{1}{x}$, (ii) $\dfrac{2a-3}{2a}$, (iii) $\dfrac{a^2-3}{a}$, (iv) $\dfrac{2a-1}{a}$. 5. 40.

130. 6. $\dfrac{2pq}{p+q}$ m.p.h. 7. $\frac{5}{2}x+\frac{5}{2}$; $x=\frac{3}{5}$, $y=-\frac{1}{5}$.

8. Temperature drops after midnight till about 3 a.m., then rises till about 4 p.m. and then drops again.

From 12.30 p.m. to 8.30 p.m.

Paper 21

1. -6, -24, 0; 0, 1. 2. $x+5y-6$.
3. $\frac{7}{5}$. 4. $-\dfrac{x+y}{6}$, $\dfrac{3W-90}{10}$.
5. £200 at 4 %, £800 at 5 %.
6. $a-2x$, $b-2x$, $c-2x$ in.;

(i) abc, (ii) $(a-2x)(b-2x)(c-2x)$,

(iii) $abc-(a-2x)(b-2x)(c-2x)$ cu. in.

131. 8. (i) 100 million units, 15,500 customers, (ii) number of customers has gone up steadily; number of units has always gone up but the rate of increase slowed down in 1930, 1931.

Paper 22

1. $-\frac{47}{15}$. 2. (i) $2b-2a$, (ii) 0, (iii) $a-90$. 3. $-6\frac{3}{10}$.

132. 4. $\dfrac{10y-x}{12}$, (i) $\dfrac{10-x}{12}$, (ii) $\dfrac{10p-q}{12}$.

5. 7. 6. (ii) $bc(a-2b)$.

8. In general, when yield goes up, price goes down and vice versa; this was not the case in 1923, 1929, 1931.

[*continued overleaf*